Newton's Law of Cooling

University of Minnesota

Preliminaries and Objectives

Preliminaries:

- Standard model for exponential growth and decay

$$A(t) = A_0 e^{rt}$$

- Conversion between logaritmic form and exponential form

Objectives:

- Solve problems using Newton's Law of Cooling

Cooling to Room Temperature

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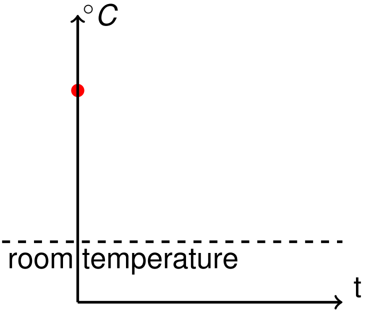

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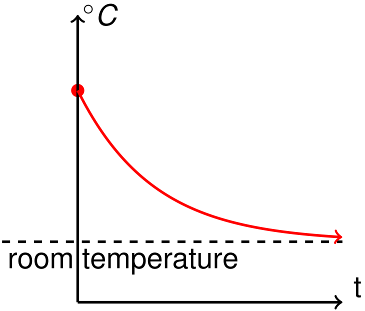

Notation

$$T^*(t) = T_0^* e^{-kt}$$

- $T^*(t) =$ temperature at time t
- $T_0^* =$ initial temperature
- $t =$ time
- $k =$ rate of decay

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Example

A cup of coffee is heated to $80^\circ C$. It is placed in a room whose temperature is $20^\circ C$. After 2 minutes, the temperature has decreased to $60^\circ C$. At what time will the coffee have cooled to $40^\circ C$?

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Time	Temp
0	80
2	60
?	40

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$$40 = 60e^{-2k} \Rightarrow k \approx 0.20273$$

$$20 = 60e^{-0.20273t} \Rightarrow t \approx 5.42 \text{ min.}$$

Recap

Newton's Law of Cooling

$T^*(t) = T_0^* e^{-kt}$

All temperatures are measured as temperature above room temperature.

Details of Example

$$T^*(t) = T_0^* e^{-kt}$$

$$\Rightarrow 40 = 60e^{-2k}$$

$$\Rightarrow \frac{40}{60} = e^{-2k}$$

$$\Rightarrow \ln \frac{40}{60} = -2k$$

$$\Rightarrow \ln \frac{60}{40} = 2k$$

$$\Rightarrow k = \frac{\ln 3/2}{2}$$

$$\Rightarrow k \approx 0.20273$$

Details of Example

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$$\Rightarrow 20 = 60e^{-0.20273t}$$

$$\Rightarrow \frac{20}{60} = e^{-0.20273t}$$

$$\Rightarrow \ln\frac{20}{60} = -0.20273t$$

$$\Rightarrow \ln 3 = 0.20273t$$

$$\Rightarrow t = \frac{\ln 3}{0.20273}$$

$$\Rightarrow t \approx 5.42$$