1. The Domain and Range of a Function

2. You should be familiar with the concept of a mathematical function. In this lesson, we will discuss which numbers are allowed as inputs to various functions, and as outputs of various functions.

3. For many functions dealing with real numbers, we have no concerns about which numbers can be used as inputs to the function. For example, for the function $f(x) = x + 3$, the function which adds three to a number, we can use any value for x.

4. The list of all possible inputs to a function is called the **domain** of the function.

5. For the function $f(x) = x + 3$, the domain is all real numbers. In text books, a bold-face $\mathbb{R}$ is used to denote all real numbers. To draw one by hand, draw a double vertical line at the left to begin.

6. Occasionally, we need to be careful about what we plug into a function. The first example is division. For the function $f(x) = \frac{1}{x}$, we can't divide by zero. In this case, we need to restrict the domain so that the number 0 can't be used as an input for the function.

7. The domain of the function $f(x) = \frac{1}{x}$ is all real numbers except zero. In the formal set builder notation, the domain is the set of all real numbers x such that x is not equal to zero. The vertical bar stands for 'such that'.

8. Another common example is the square root function. We cannot plug negative numbers into the square root function.

9. Formally, we say that the domain is all real numbers greater than or equal to zero.

10. (a) Occasionally, the function is a bit more complicated, but the same principles apply. For this function, the denominator cannot be zero. But the denominator is more complicated than the single variable x.
    (b) We need to solve an equation. The denominator, $x - 3$, cannot equal zero.
    (c) Therefore, x cannot equal 3.

11. (a) Here is another example. We cannot take the square root of a negative number, so the quantity that we are taking the square root of, $2x + 8$,
    (b) must be greater than or equal to zero. We then solve this inequality for x.
    (c) .

12. A related idea is that of the **range** of a function. The range is the list of all possible outputs of a function. In other words, the range is the set of all y-values.

13. (a) As an example, let's examine the function $y = x^2$. The vertex is at the origin, and both branches point upward.
    (b) We can think of the range as the portion of the y-axis on which the graph casts its shadow. If we move left and right from the graph, we see which portion of the y-axis is hit.
    (c) In this case, the range is all y-values greater than or equal to zero.

14. Here is another example, the absolute value function. Once again, the graph points upward from the origin, so the range is all y-values greater than or equal to zero.

15. The square root graph is an example where both the domain and range are restricted.

16. Here is an example where the parabola has been flipped top to bottom, then shifted 1 right and 2 down. In this case, the vertex has a y-value of -2 and the parabola goes downward from there.

17. To recap, there are functions for which we must be careful about which numbers we use as inputs, the most common of which are square roots and fractions. The list of all possible inputs to a function is called the domain of the function. The list of all possible outputs is called the range of the function.