1. Scientific Notation

2. You should be familiar with exponential notation and the Laws of Exponents. In this lesson, we will define scientific notation and perform arithmetic operations using scientific notation and significant digits.

3. (a) Some numbers that we deal with are of a reasonable size. For example, there are approximately 39.37 inches in a meter. We generally have a good understanding of how large a number close to 40 is.
   (b) Other numbers are far too large to have a good sense of their size. Avogadro's number counts the number of molecules in a mole of a particular substance. It is difficult to know what this number is called, and we don't want to have to write down all of those zeroes; it would be good to have a shortcut.
   (c) Likewise, some numbers are very small. We have a hard time understanding exactly how short the wavelength of the color red in the rainbow is.

4. (a) To shorten the amount we have to write, we start by counting how many digits are in the number we are representing. We will use a power of ten close to the given number.
   (b) If we take 10 to the 23rd power, we get a 1 followed by 23 zeroes. Avogadro's number is a 6 followed by 23 other digits, and therefore is slightly larger than 10^{23}.
   (c) If we move the decimal point 23 places to the left, so that it leaves only a single digit to the left of the decimal point, we arrive at scientific notation. The original number is expressed as a number between 1 and 10, multiplied by 10 to some power.

5. This is scientific notation. Every real number can be expressed as a number n, between 1 and 10 multiplied by an integer power of 10. The number n will have a single digit to the left of the decimal. The power of 10 counts how far and in which direction to move the decimal to return to the standard representation of the number. The exponent on 10 is called the **magnitude** of the number.

6. (a) Returning to some examples. Avogadro's number has 23 digits after the initial digit, and is represented as 6.022×10^{23}.
   (b) The wavelength of red in the visible light spectrum is 0.000074 millimeters. To express this in scientific notation, we need to move the decimal point until a single non-zero digit is to the left of the decimal. In this case, we move the decimal until it is to the right of the 7. We moved the decimal point five spots, so the scientific notation is 7.4×10^{-5}
   (c) In this third example, we move the decimal to the left 10 places. The scientific notation is 2.65×10^{10}.
   (d) In the final example, we move the decimal to the right 8 places. The scientific notation is 1.74×10^{-8}.

7. (a) When adding numbers in scientific notation with the same power of 10, we can add the numbers on the left in the usual fashion. This problem asks us to add 1.74 billion and 2.49 billion, where the billion is 10^9. We can think of 10^9 in the same way we think of a variable like y. When adding $1.74y$ and $2.49y$, we add the numbers which are still multiplied by y.
   (b) The same is true here, add 1.74 and 2.49 to get 4.23 which is then multiplied by 10^9.
8. (a) When adding numbers in scientific notation with different powers of 10, there are no special rules.
   (b) Our only recourse is to convert the numbers back to standard notation
   (c) and add them in the usual manner.
9. (a) In mathematics, we assume numbers are precisely as written. Scientists however are making real-world measurements that have some margin of error and only express the digits of which they are certain. The scientific notation 1.74×10^9 expresses that the measurement is correct for the digits that are listed, but there may be other digits after the '4', we just aren't certain what they are.
   (b) The three digits in red are correct, and may be followed by seven zeroes,
   (c) or may be followed by some other seven digits
   (d) or may have been followed by digits that would round up to the given scientific notation.
10. (a) When adding in this case, we should be careful to only use the precision indicated by the scientific notation.
    (b) First we convert to the standard representation. The significant digits are marked in red.
    (c) Then we add as usual.
    (d) Since the number on the top line is only correct to the nearest 10 million, we should ignore the rightmost seven digits in the addition, and stop at the decimal place to which the top number is accurate. In general, find the digits that are significant on each line, and exclude those digits that are further right than the rightmost significant digit on any particular line.
    (e) The scientific notation should include only three significant digits.
11. (a) Here is a second example.
    (b) In this case, the first two digits on the bottom line will be significant in the sum.
    (c) The numbers are added as usual. Only the first three digits are significant, the last seven digits are not, so we should round the answer to the nearest ten million.
    (d) The scientific notation contains three significant digits.

12. (a) When multiplying, we can rearrange the factors to multiply in any order.
    (b) We can first multiply 3.71 and 6.4. We can then multiply the powers of 10. Recall that when multiplying numbers, we add exponents.
    (c) To get to scientific notation, we need to make sure we have only one digit to the left of the decimal, so we move the decimal point one place to the left, and adjust the power of 10 by one factor of 10.
    (d) If we are concerned with significant digits, we should round so as to have only two significant digits. One of the initial factors had only two significant digits, so the product may not be accurate after the second significant digit. In general, the number of significant digits in the answer should be the smallest number of significant digits in any of the factors.

13. To recap: Scientific notation expresses any number as a single digit number times a power of 10. When adding, do not include any digits as significant unless they are significant in all of the numbers being added. When multiplying, the number of significant digits in the product is the smallest number of significant digits in any factor.