1. Sequences

2. You should be familiar with the concept of recursion. In this lesson, we give develop some notation that is commonly used when describing sequences of numbers.

3. Many patterns in mathematics are represented by a list of numbers. There is a first number, a second number, and so on. Typically, the list is a sequence of blanks, read from left to right.

4. There are other ways to represent the same information. A table has the same numbers listed in the right column associated with their location in the list. The advantage of a table is that you can skip several spots and still know how far you are down the sequence. The indices in the left column could also simply be numbers. This is more natural if you are trying to describe a sequence by an equation where the number in the left column is the input value.

5. A third way to represent a sequence is by a label with subscripts. This sequence is the sequence of Fibonacci numbers and we will use the capital letter F to denote terms in the sequence. The first term $F_1 = 1$. The second term F_2 also equals 1. Every successive term is found by adding the previous two terms.

6. If we wish to refer to the entire sequence all at once, we use the set notation with curly braces. This symbol stands for the entire set of numbers in the sequence F.

7. Some sequences have easy formulas. For example, here is the sequence of positive even numbers. It is easy to see that the value is twice the term's position. This is one way of defining a sequence, with an explicit formula. A term in the sequence can be found directly by plugging in a value for n. For example, the 20th even number is 40.

8. Another way to define a sequence is with a recursive formula. Here are the positive odd numbers. We begin by specifying the first term. The first odd number is 1. We then specify how to get from one term to the next. To get the next odd number, we add 2.

9. This can also be done with subscripts. For any term D_n, we find the next term D_{n+1} by adding 2.

10. The recursion formula can be more complicated. Here is the recursive formula for the Fibonacci sequence.

11. To recap: Sequences are an ordered list of numbers. One way to define a sequence is by an explicit formula. Sequences can also be defined by a recursive formula. Terms in the sequences are denoted using a subscript to determine their position.