Optimization Using Parabolas

University of Minnesota

Preliminaries and Objectives

Preliminaries

- Parabolas
- Axis of Symmetry
- Vertex
- Graph Transformations

Objectives

- Find the maximum or minimum value of a quadratic equation

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Axis of Symmetry and Vertex

$$y = ax^2 + bx + c$$

Axis of symmetry

$$x = -\frac{b}{2a}$$

(This is also the x-coordinate of the vertex)

To find the y-coordinate of the vertex, plug the x-coordinate into the original equation.

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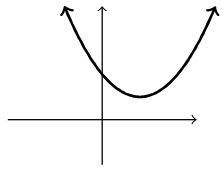
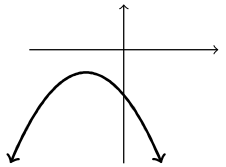
Opens Up / Opens Down

If $a > 0$, the vertex is the minimum



If $a < 0$, the vertex is the maximum



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Credits

Written by: Mike Weimerskirch

Narration: Mike Weimerskirch

Graphic Design: Robert Hank

Example 2

100 feet of fence is to be used to build a rectangular pen. What dimensions will give the maximum area?

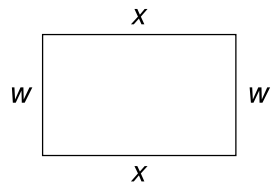
$$2x + 2w = 100$$

$$w = 50 - x$$

$$\text{Area } = x(50 - x) = 50x - x^2$$

The maximum will occur at the vertex of the parabola $50x - x^2$, so

$$x = -\frac{50}{2(-1)} = 25$$



Therefore $w = 50 - 25 = 25$ and the area of the pen is $(25)(25) = 625$ square feet.