

Completing the Square



Preliminaries and Objectives

Preliminaries

- Expanding binomials like $(x + a)^2$
- General form of a circle, ellipse, parabola, hyperbola

Objectives

- Complete the square

Standard Form of Conic Sections

Standard form of a circle: $(x - h)^2 + (y - k)^2 = r^2$

Standard form of a vertical parabola: $y - k = \pm a(x - h)^2$

Standard form of an horiz. parabola: $x - h = \pm a(y - k)^2$

Standard form of an ellipse: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

Standard form of a horiz. hyperbola: $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Standard form of a vertical hyperbola: $\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$

Example 1

Step 5: Write in standard form

$$y^2 + 4x = 6y - x^2 + 23$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 23 + 4 + 9$$

$$(x + 2)(x + 2) + (y - 3)(y - 3) = 23 + 13 = 36$$

$$(x + 2)^2 + (y - 3)^2 = 6^2$$

$$\text{Goal : } (x - h)^2 + (y - k)^2 = r^2$$

This is a circle with radius = 6, with center at $(-2, 3)$

Example 2

Step 5: Write in standard form

$$y + x^2 = 8x - 19$$

$$y + 19 + -16 = -x^2 + 8x + -16$$

$$y + 19 + -16 = -(x^2 - 8x + 16)$$

$$y + 19 + -16 = -(x - 4)(x - 4)$$

$$y + 3 = -(x - 4)^2$$

$$\text{Goal : } y - k = A(x - h)^2$$

This is a parabola, pointed downward, with vertex at $(4, -3)$

Example 3

Step 5: Write in standard form

$$4y^2 + 12x = 9x^2 + 4y + 39$$

$$4y^2 - 4y + 1 - 9x^2 + 12x + -4 = 39 + 1 - 4$$

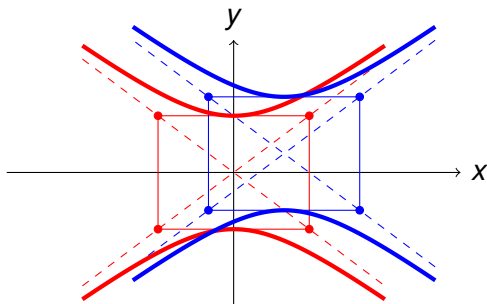
$$4\left(y^2 - y + \frac{1}{4}\right) - 9\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) = 39 + -3$$

$$4\left(y - \frac{1}{2}\right)\left(y - \frac{1}{2}\right) - 9\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = 39 + -3 = 36$$

$$\frac{\left(y - \frac{1}{2}\right)^2}{9} - \frac{\left(x - \frac{2}{3}\right)^2}{4} = 1$$

Begin with an hyperbola with the y -axis as the transverse axis. The asymptotes have slope $\pm\frac{3}{2}$. The vertices are at $(0, 3)$ and $(0, -3)$. This hyperbola is then shifted to the right $\frac{2}{3}$ and up $\frac{1}{2}$

Example 3



$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

$$\frac{(y - \frac{1}{2})^2}{9} - \frac{(x - \frac{2}{3})^2}{4} = 1$$

Completing the Square:

- Identify the general form and rearrange terms
- Factor out coefficients on x^2 and y^2
- Determine perfect square
- Supply missing constants
- Write equation in standard form