1. Root Form of a Parabola

2. You should be familiar with parabolas in both the standard and general forms and their graphs. You should also be familiar with the Factor-Root Theorem and with factoring. In this lesson, we will graph parabolas in root form and find other information about the parabola, like its axis of symmetry and its vertex.

3. (a) Recall that a parabola is a polynomial of degree two, which may appear in factored form where the leading term is x^2.
   (b) If we are given the factored form, we could expand the polynomial to get the general form. Conversely, a problem may begin with the general form and we may wish to factor the polynomial.
   (c) The advantage of factoring the polynomial is that we can now determine the roots from the Factor-Root Theorem. Since $(x-5)$ is a factor, then one of the roots is 5, and therefore 5 is an x-intercept.
   (d) From the factor of $(x+3)$, we can see the other root is at -3.
   (e) Once we have established the roots, we have a good start to graphing the parabola. Next, we can find the axis of symmetry, which will be halfway between the two roots. In this case, the axis of symmetry will be at $x = 1$. In general, you can find the axis of symmetry by averaging the roots.
   (f) If we plug $x = 1$ back into the original equation, we find the y-coordinate of the vertex to be at -16.
   (g) Once we have plotted the vertex, we can sketch the graph.

4. Formally, we say a quadratic is in root form if it is factored into a constant a, which represents the vertical stretch factor, and two linear factors which are each x minus a root. Note that if a root is negative, the factor will be x plus a constant. Roots are also x-intercepts. If a root is the input to the quadratic function, the outputted y-value will be zero. For this reason, roots are also called **zeroes** of the function. To find the axis of symmetry, find the average of the roots.

5. (a) Here is a second example. We begin by factoring out the leading coefficient, which in this case is 3.
   (b) Next, we factor what remains.
   (c) We can now see the roots are at -6 and -2,
   (d) and therefore the axis of symmetry is at -4.
   (e) Plugging -4 back into any equation finds the y-coordinate of the vertex to be -12.
   (f) We could find two additional points by using the stretch factor. A standard $y = x^2$ parabola goes left and right one from the vertex, and up 1. This parabola has a stretch factor of 3, so we go up three instead. When we move left and right one from the vertex, we should go up 3, to -9, so there will be points at $(-5, -9)$ and $(-3, -9)$. We can then sketch the parabola.

6. (a) Here is a third example. It is easiest if we factor out the negative sign,
   (b) and then factor the polynomial inside the parentheses.
   (c) The x-intercept will occur when y equals zero, which by the Zero-Product Property occurs when one of the factors equals zero.
   (d) We can solve these two equations separately to find the roots,
   (e) which will be at $3/2$ and -1.
   (f) Averaging the roots, we find the x-coordinate of the vertex to be $\frac{1}{4}$ and plugging $\frac{1}{4}$ in for x, we find the y-coordinate of the vertex to be at $\frac{25}{8}$.
   (g) We can then sketch the graph.

7. (a) As a final example, we find the vertex of a parabola expressed in root form.
   (b) The roots are at 7 and -3.
   (c) Averaging the two roots finds the x-coordinate of the vertex to be at 2.
   (d) Plugging 2 into the original equation gives the y-coordinate of the vertex to be 100.
   (e) The vertex is at $(2, 100)$, with the axis of symmetry at $x = 2$.

8. To recap: When a quadratic is factored, the roots can be determined from the factors by the Factor-Root Theorem. The axis of symmetry will be the average of the two roots.