1. Parabolas

2. You should be familiar with polynomials, the graph of the standard parabola $y = x^2$ and transformations of graphs. In this lesson, given the equation of a parabola, we will find the axis of symmetry and the vertex, and graph the parabola.

3. A quadratic function is a polynomial of degree 2. It's graph is called a parabola.

4. The standard parabola, $y = x^2$ is symmetric with respect to the y-axis. The y-axis divides the parabola into two identical branches. The y-axis is therefore the axis of symmetry. The vertex of the parabola is at the origin. To find other points on the parabola, we go over 1, up 1, and over 2, up 4.

5. (a) Now let's examine the general parabola. Adding c will move the graph up and down, so we can save this step until the end.
   (b) The standard parabola is x^2 without the factor of a, which is causing a stretch. If we factor out the a,
   (c) we can adjust the parabola by the stretch factor as the last step.
   (d) We can first concentrate on what this parabola looks like
   (e) by factoring out an x, we can find the axis of symmetry.

6. (a) In factored form, it is easy to find the roots of a polynomial. In this case, there is an x-intercept of 0, and an x-intercept of $-b/a$. That is to say, if we plug in 0 for x, then y will be zero and the origin will be a point on the graph.
   (b) If $x = -\frac{b}{a}$, then the second factor will be zero and once again, y will be zero, so $-\frac{b}{a}$ is an x-intercept.
   (c) Since the parabola is symmetric left and right, the axis of symmetry must be halfway between these two points, that is, the axis of symmetry must be at $x = -\frac{b}{2a}$.
   (d) The x-coordinate of the vertex, must lie on the axis of symmetry.
   (e) This will be true for this parabola, which we can rewrite in expanded form
   (f) The x-coordinate of the vertex will not change if we stretch vertically by a factor of a
   (g) Nor will it change if we shift the graph vertically by c
   (h)

7. Now that we now the axis of symmetry, and therefore the x-coordinate of the vertex, we can find the y-coordinate of the vertex by plugging the x-coordinate back into the original equation. Finding the y-coordinate of the vertex also accomplishes the vertical shift, so all that remains is the vertical stretch.

8. (a) We can now use this procedure to graph a parabola from its equation. First, we find the axis of symmetry, which is at $x = -\dfrac{b}{2a}$.
   (b) This also gives us the x-coordinate of the vertex.
   (c) To find the y-coordinate of the vertex, plug the x-value 2 back into the original equation. We get the y-coordinate of the vertex to be -5.
   (d) We now are ready to graph the parabola.
   (e) The vertex is at $(2, -5)$. The standard parabola goes over 1, up 1. This parabola has a stretch factor of 3, so we should go over 1, up 3
   (f) The standard parabola goes over 2, up 4. This parabola has a stretch factor of 3, so we should go over 2, up 12.
   (g) Here is the graph of the parabola.

9. To recap: The axis of symmetry, and x-coordinate of the vertex is $x = -\dfrac{b}{2a}$. The y-coordinate of the vertex can be found by plugging the x-value back into the original equation. The parabola is then stretched vertically by a factor of a.