1. Optimization Using Parabolas

2. You should be familiar with parabolas and their graphs, as well as graph transformations. In this lesson, we will find maximum and minimum values of functions.

3. Recall that the x-coordinate of the vertex of a parabola is $-\dfrac{b}{2a}$. The y-value can be found by plugging the x-value into the original function.

4. If the coefficient on x^2 is positive, then the parabola opens upward, and the vertex is the minimum. If the coefficient on x^2 is negative, then the parabola opens downward, and the vertex is the maximum.

5. (a) Here is an example of a typical application. A farmer wishes to maximize profit. The profit function is given, and the coefficient on x^2 is negative, so the parabola opens downward.
   (b) The vertex will be the maximum. The x-coordinate of the vertex is at $-\dfrac{b}{2a}$, which in this case is 5.5.
   (c) The profit is found by plugging 5.5 into the original equation.
   (d) The maximum profit to the farmer is $8250

6. (a) In this example, we will build a fence that is x feet long and w feet wide. The constraint is that we only have 100 feet of fence and we wish to build the rectangle with the largest area.
   (b) Since we have 100 feet of fence, $2x + 2w = 100$.
   (c) Solving for w, we get $w = 50 - x$
   (d) Therefore the area is given by the function $50x - x^2$
   (e) The maximum occurs at $-\frac{b}{2a}$, which in this case is when $x = 25$. The pen should be 25 feet long, and therefore also 25 feet wide. The total area will be 625 square feet.

7. To recap: If a is positive, the vertex will be the minimum point on the parabola. If a is negative, the vertex will be the maximum point of the parabola.