1. The Factor-Root Theorem

2. You should be familiar with polynomials, factoring and function notation. In this lesson, we will define what it means to be a **root** of a polynomial, and use the connection between roots and factors to factor polynomials.

3. (a) We start first with the Zero-Product Property which discusses when the product of two numbers, x and y can equal zero.
   (b) It is easy to see that if $x = 0$, then the product is zero. Likewise if $y = 0$, the product is zero.
   (c) If you multiply two non-zero numbers, the answer will not be zero.
   (d) Therefore, if the product of x and y is zero, then either $x = 0$ or $y = 0$.

4. (a) We now define what it means to be a **root** of a polynomial. A root of a polynomial is a number that when substituted for x, gives the answer 0. In other words, r is a root, if when r is used as the input to the polynomial function, then the output is zero.
   (b) In terms of the graph of the polynomial, when the output is zero, the $y-$value is zero, which occurs at the $x-$intercepts. Therefore roots are synonymous with $x-$intercepts.
   (c) Returning to the equation, to figure out if 3 is a root of the polynomial $p(x)$,
   (d) simply substitute 3 for x and calculate the answer.
   (e) In this case, the answer is zero,
   (f) so 3 is a root of the polynomial p.

5. (a) Here is an example for you to try. Is 1 a root of the polynomial? Is 3 a root of the polynomial? You may wish to pause the video to work out these answers.
   (b) When 1 is plugged into the function, the answer is 2, so 1 is not a root. 3 is a root, since we get zero as the answer.

6. (a) Taking another look at the previous example, we could rewrite the polynomial in factored form.
   (b) We can now see that if we plug in 2, then $p(x)$ will equal zero, because one of the factors is zero. Likewise, if we plug in 3, we will get something times zero, which is zero.
   (c) Using the Zero-Product Property, once we have a polynomial in factored form, we can solve easier equations. If $p(x) = 0$, then one of the factors of $p(x)$ must also equal zero and we can solve those equations instead.

7. (a) When the factor is a simple linear factor, it is easy to solve. If $(x - r)$ is a factor, then substituting r will give a factor of zero, and therefore $p(x) = 0$ and r will be a root.
   (b) The converse is harder to see, but is also true. If r is a root of a polynomial, then $(x - r)$ is a factor. We won't go into the details of why this is true here, other than to state that if you performed the long division of polynomials, there would be no remainder.

8. (a) This second fact, that roots can help us find factors can be useful in factoring polynomials. First we need need some way to find a root. One possibility is to use a graphing utility like Desmos.com.
   (b) If we graph the polynomial, we can look for the $x-$intercepts. In this case, the graph crosses the $x-$axis at $x = 4$, so 4 is a root, and $(x - 4)$ is a factor.
   (c) This let's us get started on the factoring. We know one of the factors is $(x - 4)$.
   (d) Since the leading term is x^3, we will need the second factor to contain x^2, so that when we multiply by x, we get x^3.
   (e) We can find the constant term in a similar manner. We need to find the number which we can multiply by -4 to get -12.
   (f) That number is 3. In order to find the middle term, it takes a bit more work. One way to get there is to notice that the term $-x^2$ is going to come from two places; $-4x^2$ and the product of the missing term with x. We want $-x^2$, we have $-4x^2$, so we will need to add $3x^2$, which means that the missing term is $+3x$.
   (g) To verify that this factorization is correct, we can expand the right side.

9. (a) Here is a final example for you to try. You are given a list of possible roots. Plug each potential root into the polynomial $p(x)$ to see which numbers are roots. Once you have found the roots, you will know the factors. You may wish to pause the video to work out the answer.
   (b) There are three roots, -4, -2 and 1. Therefore the three factors are $(x + 4)$, $(x + 2)$, and $(x - 1)$.

10. To recap: To test if a number r is a root of a polynomial, plug r into the function and test to see if the result is zero. If r is a root of the polynomial, then $(x - r)$ is a factor.