The Factor-Root Theorem

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Preliminaries and Objectives

Preliminaries

- Polynomials
- Factoring
- Function Notation

Objectives

- Definition of **root**
- Factor polynomials

Zero Product Property

$$(x)(y) = 0$$

$$(0)(y) = 0$$

$$(x)(0) = 0$$

If $x \neq 0$ and $y \neq 0$, then $xy \neq 0$

Zero Product Property

If $xy = 0$, then $x = 0$ or $y = 0$

Definition of Root

Let $p(x)$ be a polynomial function, then r is a **root** of p if $p(r) = 0$

Example: Is $x = 3$ a root of $p(x) = x^2 - 8x + 15$?

Plug in the value to find $p(3) = 3^2 - 8(3) + 15 = 0$

Since $p(3) = 0$, then $x = 3$ is a root of $p(x)$

Example 1

$p(x) = x^2 - 5x + 6$

Is $x = 1$ a root?

$p(1) = 1^2 - 5(1) + 6 = 2$ No, $x = 1$ is not a root.

Is $x = 3$ a root?

$p(3) = 3^2 - 5(3) + 6 = 0$ Yes, $x = 3$ is a root.

Example 1

$p(x) = x^2 - 5x + 6$

$p(x) = (x - 2)(x - 3)$

If $x = 2$, then $p(x) = (2 - 2)(\ldots) = 0$
If $x = 3$, then $p(x) = (\ldots)(3 - 3) = 0$

If $p(x) = (x - 2)(x - 3) = 0$, then $x - 2 = 0$ or $x - 3 = 0$

Factor-Root Theorem

Factor-Root Theorem

- **Part I -** If $p(x)$ is a polynomial and $(x - r)$ is a factor of $p(x)$, then r is a root.
- **Part II -** If r is a root of $p(x)$, then $(x - r)$ is a factor of $p(x)$.

Example 2

Factor $p(x) = x^3 - x^2 - 9x - 12$

$x = 4$ is a root since $p(4) = 4^3 - 4^2 - 9(4) - 12 = 0$, therefore $(x - 4)$ is a factor.

$p(x) = x^3 - x^2 - 9x - 12 = (x - 4)(x^2 + 3x + 3)$

Example 3

Let $p(x) = x^3 + 5x^2 + 2x - 8$.

Check to see which of the following are roots.
$\{-4, -2, -1, 1, 2, 4\}$.

Factor $p(x)$.

The three roots are -4, -2 and 1, therefore
$p(x) = (x+4)(x+2)(x-1)$

Recap

Definition of Root

Let $p(x)$ be a polynomial function, then r is a **root** of p if $p(r) = 0$

Factor-Root Theorem

- **Part I -** If $p(x)$ is a polynomial and $(x - r)$ is a factor of $p(x)$, then r is a root.
- **Part II -** If r is a root of $p(x)$, then $(x - r)$ is a factor of $p(x)$.