

Fractions within Fractions



Preliminaries

- Factor polynomials
- Adding fractions
- Multiplying and dividing fractions
- Reducing fractions

Objectives

- Simplify fractions within fractions

$$\frac{\frac{x+1}{x-1} + \frac{x+2}{x}}{\frac{x+4}{x^2-1} - \frac{3x}{x-1}}$$

Method 1 - Separate numerator and denominator

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Method 2 - Find common denominator and clear fractions

Numerator:

$$\begin{aligned} \frac{x+1}{x-1} + \frac{x+2}{x} &= \frac{x(x+1)}{x(x-1)} + \frac{(x+2)(x-1)}{x(x-1)} \\ &= \frac{x^2+x}{x(x-1)} + \frac{x^2+x-2}{x(x-1)} = \frac{2x^2+2x-2}{x(x-1)} \end{aligned}$$

Denominator:

$$\begin{aligned} \frac{x+4}{x^2-1} - \frac{3x}{x-1} &= \frac{x+4}{(x+1)(x-1)} - \frac{3x}{x-1} \\ &= \frac{x+4}{(x+1)(x-1)} - \frac{3x(x+1)}{(x-1)(x+1)} = \frac{x+4}{(x+1)(x-1)} - \frac{3x^2+3x}{(x-1)(x+1)} \\ &= \frac{-3x^2-2x+4}{(x-1)(x+1)} \end{aligned}$$

$$\begin{aligned} \frac{2x^2+2x-2}{x(x-1)} \div \frac{-3x^2-2x+4}{(x-1)(x+1)} &= \frac{2x^2+2x-2}{x(x-1)} \cdot \frac{(x-1)(x+1)}{-3x^2-2x+4} \\ &= \frac{2x^2+2x-2}{x} \cdot \frac{(x+1)}{-3x^2-2x+4} \end{aligned}$$

$$\begin{aligned} \frac{\frac{x+1}{x-1} + \frac{x+2}{x}}{\frac{x+4}{x^2-1} - \frac{3x}{x-1}} &= \frac{\frac{x(x+1)(x-1)}{x(x-1)} + \frac{x(x+2)(x-1)}{x(x-1)}}{\frac{x(x+1)(x+1) + (x+2)(x+1)(x-1)}{x(x+4) - 3x(x+1)}} \\ &= \frac{x^3+2x^2+x+x^3+2x^2-x-2}{x^2+4x-3x^3-3x^2} \\ &= \frac{2x^3+4x^2-2}{-3x^3-2x^2+4x} \end{aligned}$$

Recap

- Approach 1: Simplify numerator and denominator separately
- Approach 2: Clear smaller fractions by multiplying by all of the factors.