

Fractions within Fractions



Preliminaries and Objectives

Preliminaries

- Factor polynomials
- Adding fractions
- Multiplying and dividing fractions
- Reducing fractions

Objectives

- Simplify fractions within fractions

Method 1 - Separate numerator and denominator

$$\frac{x+1}{x-1} + \frac{x+2}{x}$$

$$\frac{x+4}{x^2-1} - \frac{3x}{x-1}$$

Method 1 - Separate numerator and denominator

Numerator:

$$\begin{aligned}\frac{x+1}{x-1} + \frac{x+2}{x} &= \frac{x(x+1)}{x(x-1)} + \frac{(x+2)(x-1)}{x(x-1)} \\ &= \frac{x^2+x}{x(x-1)} + \frac{x^2+x-2}{x(x-1)} = \frac{2x^2+2x-2}{x(x-1)}\end{aligned}$$

Denominator:

$$\begin{aligned}\frac{x+4}{x^2-1} - \frac{3x}{x-1} &= \frac{x+4}{(x+1)(x-1)} - \frac{3x}{x-1} \\ &= \frac{x+4}{(x+1)(x-1)} - \frac{3x(x+1)}{(x-1)(x+1)} = \frac{x+4}{(x+1)(x-1)} - \frac{3x^2+3x}{(x-1)(x+1)} \\ &= \frac{-3x^2-2x+4}{(x-1)(x+1)}\end{aligned}$$

Method 1 - Separate numerator and denominator

$$\begin{aligned} & \frac{2x^2 + 2x - 2}{x(x - 1)} \div \frac{-3x^2 - 2x + 4}{(x - 1)(x + 1)} \\ &= \frac{2x^2 + 2x - 2}{x(x - 1)} \cdot \frac{(x - 1)(x + 1)}{-3x^2 - 2x + 4} \\ &= \frac{2x^2 + 2x - 2}{x} \cdot \frac{(x + 1)}{-3x^2 - 2x + 4} \end{aligned}$$

Method 2 - Find common denominator and clear fractions

$$\begin{aligned} & \frac{\frac{x+1}{x-1} + \frac{x+2}{x}}{\frac{x+4}{x^2-1} - \frac{3x}{x-1}} \cdot \frac{\frac{x(x+1)(x-1)}{1}}{\frac{x(x+1)(x-1)}{1}} \\ &= \frac{x(x+1)(x+1) + (x+2)(x+1)(x-1)}{x(x+4) - 3x(x)(x+1)} \\ &= \frac{x^3 + 2x^2 + x + x^3 + 2x^2 - x - 2}{x^2 + 4x - 3x^3 - 3x^2} \\ &= \frac{2x^3 + 4x^2 - 2}{-3x^3 - 2x^2 + 4x} \end{aligned}$$

Recap

- Approach 1: Simplify numerator and denominator separately
- Approach 2: Clear smaller fractions by multiplying by all of the factors.