1. Factoring: The AC-method for Factoring Trinomials

2. You should be familiar with the distributive property and with expanding binomials. This technique relies on factoring by grouping. We will combine this factoring technique with other factoring techniques, including greatest common factor and difference of squares.

   In this lesson, we will factor trinomials, that is, we will factor polynomials with three terms by splitting apart the middle term to create a factoring by grouping problem.

3. (a) Recall the FOIL process for expanding binomials. We begin by multiplying the first terms, then the outside terms, then the inside terms, and finally the last terms. This produces a four part answer.

   (b) We then combine like terms to simplify.

   (c) To factor, we begin with the trinomial and wish to produce the factors. An important step will be to reproduce the outside term and inside term that got combined. The four term expression came from the FOIL process. The three term expression has the general form $Ax^2 + Bx + C$. We need to split the Bx term into the Outside term and the Inside term.

   (d) We begin with a diagram in which we place the coefficieint B at the bottom. We know the outside term and inside term will add to Bx. We need one other piece of information, which is the product of the inside and outside coefficients, in this case, -120. But if we don't know the inside and outside terms, how will we know this product? The answer is that the product can be arrived at in a second way. Notice that the 15 came from multiplying 5 and 3 and the -8 came from multiplying the 2 and the -4. When we multiply 15 by -8, we have multiplied all four of the original coefficients.

   (e) We can create this number a second way. When we find A for the first term, we multiply the leading coefficients 5 and 2. When we find the last term, we multiply -4 and 3 to get C. The product of A and C produces the same answer -120. We can get this information directly from the trinomial by multiplying 10 and -12.

   (f) Once we have the target product -120 and target sum 7, we try to produce two numbers which multiply to -120 and add to 7.

   (g) Recall that when the product of two integers is negative, it can only occur by multiplying one positive and one negative, so that when they are added, we are adding one positive and one negative, or in other words, we are subtracting. If the sum is positive, then the larger number will be positive and if the sum is negative, then the larger term is negative. If the product is positive, then either both terms are positive, in which case we get a positive sum. If we have a negative sum, then each term was negative.

   (h) Returning to the question at hand, we wish to factor 120, which could be 1 times 120. One hundred twenty is even, so it could be 2 times 60. One hundred twenty is also 3 times 40, 4 times 30, 5 times 24, 6 times 20. and 8 times 15. We are looking for the factor pair whose difference is 7. Since 15 is seven more than 8, the two terms we desire are 15 and -8.

(i) We now have reduced this problem to a factoring by grouping question. We fill in the grid with the four terms. The $10x^2$ and the -12 are placed. The $7x$ splits into $15x$ and $-8x$ according to the diagram at the left. We now begin anywhere we wish, this time I will start with the left hand column. The common factor is $2x$. What times $2x$ is $10x^2$? The answer is $5x$. What times $2x$ is $-8x$? The answer is -4. What times $5x$ is $15x$? The answer is 3. Finally , we verify -4 times 3 is -12 as a double-check.

(j) The factors are $(5x - 4)(2x + 3)$.

4. (a) Here is another example. First, identify the coefficients A, B and C.

   (b) Place B at the bottom.

   (c) Multiply A and C and place the product 24 at the top.

   (d) Factor 24 as 1 times 24, 2 times 12, 3 times 8, or 4 times 6.

   (e) Since the product is positive and the sum is negative, we are looking for two negative numbers that add to -11. The pair of factors 3 and 8 add to 11, so we want -3 and -8.

   (f) This allows us to complete the grid, find the common factor of the top row, find the missing factors for the columns, and the bottom row and verify the last entry.

   (g) The factors are $(x - 4)(2x - 3)$.

5. (a) Here is a third example. You may wish to pause the video to work out this answer.

   (b) First, identify the coefficients A, B and C.

   (c) Place B at the bottom.

   (d) Multiply A and C and place the product at the top.

   (e) Factor 42.

   (f) Since the produce is positive and the sum is positive, we are looking for two positivee numbers that add to 23.

   (g) This allows us to complete the grid and find the factors.

6. (a) Here is a fourth example for you to work out. You may wish to pause the video.

   (b) Place B at the bottom, multiply A and C and place the product at the top.

   (c) Factor -200.

   (d) Since the produce is negative and the sum is positive, we are looking for two numbers that subtract to 10.

   (e) The factors are $(4x - 5)(2x + 5)$.

7. To recap: A four term polynomial which has been produced by expanding two binomials can be groups into rows and columns so that the common factors can found to produce the factors. Take care to remove any common factors at the beginning. After factoring by grouping, there may be other types of factoring.

8. (a) Here is one more example which combines several factoring techniques. You may wish to pause the video to work out this problem.
   (b) First, factor out the common factor.
   (c) Next, factor the trinomial using the AC-method
   (d) Finally, use the difference of squares.