

1. Rational Exponents
2. You should be familiar with exponents, including zero exponents and negative exponents and laws for multiplying and dividing expressions with exponents. In this lesson, we will define what we mean when an exponent is a fraction, called a rational exponent.
3. (a) Can we make sense of an expression like $9^{1/2}$?
(b) We know what it means when the exponent is an integer. 9^3 is three factors of nine multiplied together, but what do we mean by half of a factor of 9?
(c) One approach is to collect enough factors and use the addition rule for exponents. Since two copies of $9^{1/2}$ multiply to make 9, then $9^{1/2} = 3$. In other words, $9^{1/2} = \sqrt{9}$.
4. (a) Though perhaps we should be a little more careful. In the real numbers, there are two square roots of 9, one positive, one negative. Do we want to include both?
(b) Also, when dealing with real numbers, we can't take the square root of a negative number. If we want an answer, we would need to use complex numbers. If you wish to learn more about complex numbers, there are other resources that will demonstrate the useful properties. In this lesson, we will stick to just using the real numbers. So how do we avoid these problems? The answer is that even the real numbers are too big of a set to deal with. However, if we use only the positive real numbers, these two problems go away. We will not try to take roots of negative numbers, and we will not use negative numbers as answers.
5. (a) The exponent $1/3$ means cube root, and so on.
(b) If the numerator of the exponent is not one, we raise the root to that power. For example, $8^{2/3}$ can be thought of as a two step process, first taking the cube root, then squaring to get the final answer. The order can be reversed, we could first square, then take the cube root. Usually, the first method will be easier since the numbers we will be working with will be smaller.
6. To recap: For a rational exponent, the numerator is a power in the ordinary sense, the denominator requires us to take a root.