

1. Parallel and Perpendicular Lines

2. You should be familiar with the slope-intercept form of a line and the point-slope form of a line. In this lesson, you will learn how to find the equation of a line through a given point parallel to a given line and perpendicular to a given line.

3. Suppose we are given the equation of line, for example $y = \frac{1}{2}x + 2$, and we wish to find the equation of a line parallel to it. What is the slope of the lower line?

(Animation) Think of this as a race between two competitors up the y -axis, with time measured on the horizontal axis. The red competitor starts with a lead of two meters. If the slope (or speed) of the lower line is greater than $\frac{1}{2}$, that is, if the blue competitor runs faster than the red competitor, the blue competitor will catch up. This is equivalent to the lines intersecting. Similarly, if the speed of the blue competitor is slower than the red competitor, then at some time in the past, the two were at the same location and red moved ahead. The only way the two competitors will never meet is if they run at the same speed. In other words, the lines have to have the same slope.

4. This leads to one definition of parallel lines, two lines are parallel if they have the same slope.

5. (a) We can solve problems involving parallel lines. Here, we are asked to find the equation of a line parallel to a given line through the point $(2, 3)$.

(b) First, we need to find the slope of the given line. The equation is not in one of our common forms, but we can rewrite it in slope-intercept form.

(c) Solving the equation of the line for the variable y , we get the slope-intercept form. If we take the second equation and divide everything by 2, we get $y = \frac{3}{2}x - \text{some number}$. It doesn't really matter what that number is, we know the slope is $\frac{3}{2}$ from the slope-intercept form.

(d) Using the slope $\frac{3}{2}$ and the given point, we can then use the point-slope form of a line to find the answer.

6. If we know the slope between two points, what is the slope of a perpendicular line?

(Animation) When we rotate the triangle by 90° , the rise and run interchange places. The change in x for the original triangle is now the change in y for the new triangle. The old change in y is now the change in x , but pointed to the left. The new slope is the negative reciprocal of the old slope. For example, if the old slope was $\frac{3}{4}$, the perpendicular slope would be $-\frac{4}{3}$.

7. (a) We can solve problems involving perpendicular lines similar to the method used for parallel lines. Here, we are asked to find the the perpendicular to a given line through the point $(-1, 6)$.

(b) First, we need to find the slope of the given line, then find the perpendicular slope by taking a reciprocal and changing sign.

- (c) Solving the equation of the line for the variable y , we get the slope-intercept form. The slope of the original line is $3/2$, so to get the slope of the perpendicular line we change the sign to negative, then take the reciprocal.
 - (d) Using the slope $-2/3$ and the given point, we can then use the point-slope form of a line to find the answer.
8. To recap: If two lines are parallel, they have the same slope. If two lines are perpendicular, their slopes are negative reciprocals.