1. Factoring

2. You should be familiar with the Distributive Law and with the FOIL method for multiplying two binomials. In this lesson, we will factor these expressions by taking an expanded polynomial and rewriting it in terms of its factors.

3. (a) Let's review the multiplication process: First, the Distributive Law. In this example, the $3x$ is multiplied to both the x, and the 4.
   (b) $3x$ times x is $3x^2$.
   (c) $3x$ times 4 is $12x$, giving the answer $3x^2 + 12x$.
   (d) Our goal now is to figure out how to take the expanded polynomial, and return it to the initial factors that it came from.

4. (a) Here is an example of the FOIL method: First, we multiply $2x$ by $3x$ to get $6x^2$.
   (b) Next we multiply the outside terms, $2x$ and -1,
   (c) then the inside terms 3 and $3x$,
   (d) then the last terms 3 and -1.
   (e) We then combine like terms to get the final answer.
   (f) .
   (g) Once again, the goal of this lesson is to take this expanded polynomial, and figure out the initial factors.

5. (a) Here is one more example of the FOIL method; in this case, in the special difference of squares pattern. We multiply the first terms,
   (b) then the outside,
   (c) then the inside,
   (d) and the last.
   (e) Notice that the inside and the outside terms cancel,
   (f) leaving us just with the first and last terms.
   (g) Once again, we will try to take this final expanded polynomial, and return it to its initial factors.

6. (a) We will now begin to undo the multiplication performed by the distributive law, and the FOIL method. Since we are undoing multiplication, the factoring process is a type of division. We are dividing an expanded polynomial into smaller factors.

   The first technique is to factor out a greatest common factor. The distributive law multiplied a single term to a polynomial. We now wish to figure out what that single term was. We look for pieces that are common to both terms.

   We might have a factor of 2 in both terms. 12 is an even number, so $12x$ has a factor of 2, but $3x^2$ does not. You may have noticed that both terms have a factor of 3, since 12 is 3 times 4. Is there anything else? Yes, both terms have the variable x. The $3x^2$ term has two factors of x, recall that x^2 is x times x. The $12x$ term has one factor of x.

(b) Formally, we can brake down each term into its smallest possible factors, its prime factors. The coefficient can be factored into prime factors. The variable can't be factored, other than to interpret the exponent as repeated multiplication. We now look for the common factors; we see each term has a factor of 3, and a factor of x, so the common factor is $3x$.

(c) We then wish to find the other factor. Here is where the process uses division. The first term is $3x^2$, we factored out a $3x$. What remains after the factoring, that is what can we multiply $3x$ by to get $3x^2$? The factor that is left over is x. When we factor $3x$ out of $12x$, we are left with the two factors of 2, which is 4.

7. (a) Here is an example for you to try; you may wish to pause the video and work out the answer.

   (b) The common factor is 2×2.

8. (a) We will now start to undo the multiplication performed by the FOIL method. The first technique is the difference of squares. Typically, the FOIL method will produce four terms. The difference of squares has just two terms. The reason is that the two middle terms canceled. This happens when the two factors are identical, except for the plus or minus sign.

   (b) All we need to do is to recognize which term gets multiplied by itself to produce the factors.

   (c) To get $4x^2$, we multiply $2x$ by itself. To get 9, we multiply 3 by itself. If we FOILed the factored expression, the middle terms would cancel.

9. (a) Here is an example for you to try. You may wish to pause the video and work out the answer.

   (b) $16x^4$ is $4x^2$ times itself.

10. Here is another example. What can we multiply by itself to get x^3? In this case, since the exponent is odd, we can't split x^3 evenly, so this expression cannot be factored.

11. In most cases, when we use the FOIL method, the middle terms combine. We then need to figure out how to split the middle term; in this case, the $11x$. Unfortunately, there is no hard and fast rule. The best we can do is make a list of the ways the first term and the last term can be factored, and then check if the middle term is correct. In this case, 3 is prime, so 3 can only be factored as 3 times 1. It does not matter in which order we write the factors, all we have to do is figure out how to split the 4. We have two choices: either 1 times 4, or 2 times 2. But we also have a choice as to which factor, the 1 or the 4, pairs with the $3x$. Also, the last term was not 4, but instead was -4, so one of the two constants will be subtracted but which one? Unfortunately, we need to try each possibility to find the correct one.

12. There are some helpful hints to deal with the signs and the factors. If the constant term has a $+c$, then the constant term came from multiplying either two positive numbers or two negative numbers. If the two constants were positive, then the two middles will both be positive, and the bx term will be positive. If the two constants were negative, then the two middle terms will both be negative, and the bx term will be negative. If the constant term in the expanded

form is $-c$, then there will be one factor with a constant added, and the other factor with a constant subtracted. What will the middle term look like? It will depend on whether the outside term or inside term is larger.

13. There is one other helpful hint. In this problem, we have a complete list of all possible ways to factor 12x2, and all possible ways to factor 28. But how do we mix and match the correct factors? We can eliminate some possibilities. The original expression has no common factors, therefore the smaller factors also cannot have common factors: we cannot pair up even terms. So, 2x could only pair up with 1, or 7 since the other constants are even. The same is true for 4x, 6x and 12x since all contain a factor of 2. Instead of 18 possibilities to try, we can eliminate most of them, and try only the ones without common factors.

14. (a) The last technique is called grouping. It occurs when the FOIL method produces four terms, none of which can be combined. Recall, in the FOIL method, that the left hand term in the left hand factor is part of both the first and the outside, and so is a common factor of the first two terms in the FOIL process. We rely on this fact to complete the factoring.

    (b) We break the problem into two halves: first, look for a common factor among the first two terms. In this case, both $6x^3$ and $15x^2$ have a factor of $3x^2$. We factor the $3x^2$ out of the first two terms. We find that the remaining factor is $2x - 5$. We repeat this process for the second half: $8x$ and 20 both have a factor of 4. We factor out the 4, and see that the remaining factor is $2x - 5$. We now have a common factor of $2x - 5$, which we factor out.

    (c) Here is the final factorization.

15. (a) Here is an example for you to try. You may wish to pause the video and work out the answer.

    (b) The first two terms had a common factor of x^2, the second two terms had a common factor of 9, each leaving a factor of $x + 2$. Notice that the first factor is a difference of squares, and can be factored further.

    (c) More than one factoring technique may be present in any given problem.

16. To recap:

    To begin the factoring process, look for common factors, and factor out the greatest common factor.

    In what remains, if it has two terms, look for the difference of squares pattern.

    If it has three terms, it likely comes from a standard FOIL process

    If it has four terms, try factoring by grouping.