

Arithmetic Sequences and Series



Preliminaries and Objectives

Preliminaries

- Sequences defined by explicit formulas
- Sequences defined by recursive formulas
- Sigma notation
- Slope-intercept form of a line

Objectives

- Define arithmetic sequences
- Find the sum of an arithmetic series

Sequences

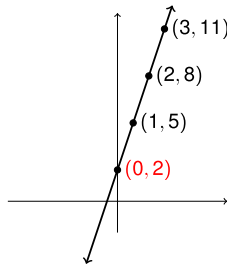
An **arithmetic** sequence is a sequence of numbers in which the recursion is to add a constant, called the *common difference*.

Explicit formula

n	0	1	2	3	4
a_n	2	5	8	11	14

As the input n changes by 1, the output a_n changes by 3

$$a_n = 3n + 2$$



Example 2

$$\{a_n\} = -7 \{-3, 1, 5, 9, 13, 17, \dots\} \quad d = 4$$

Find the n th term.

Find the 23rd term.

$$a_n = 4n - 7$$

$$a_{23} = 4(23) - 7 = 85$$

Example 3

If $a_7 = 22$ and $a_{10} = 31$, find the n th term.

$$d = \frac{31 - 22}{10 - 7} = \frac{9}{3} = 3$$

$$a_0 = 22 - 7(3) = 1$$

$$\{a_n\} = 1\{4, 7, 10, 13, 16, 19, 22, \dots\}$$

$$a_n = 3n + 1$$

Example 4

Find the sum $S = 1 + 2 + 3 + \dots + 100$

$$\sum_{n=1}^{100} n$$

$$\begin{array}{r} S = 1 + 2 + 3 \dots + 100 \\ S = 100 + 99 + 98 \dots + 1 \\ \hline 2S = 101 + 101 + 101 \dots + 101 \end{array}$$

$$2S = (101)(100)$$

$$S = \frac{(101)(100)}{2} = 5050$$

Example 5

Find the sum $S = 2 + 4 + 6 + \dots + 100$

$$\sum_{n=1}^{50} 2n$$

$$\begin{array}{r} S = 2 + 4 + 6 \dots + 100 \\ S = 100 + 98 + 96 \dots + 2 \\ \hline 2S = 102 + 102 + 102 \dots + 102 \end{array}$$

$$2S = (102)(50)$$

$$S = \frac{(102)(50)}{2} = 2550$$

Example 6

Find the sum $S = 5 + 8 + 11 + \dots + 74$

$$d = 3 \quad a_0 = 2 \quad a_n = 3n + 2 \quad 74 = 3n + 2 \quad n = 24$$

$$\sum_{n=1}^{24} 3n + 2$$

$$\begin{array}{r} S = 5 + 8 + 11 \dots + 74 \\ S = 74 + 71 + 68 \dots + 5 \\ \hline 2S = 79 + 79 + 79 \dots + 79 \end{array}$$

$$2S = (79)(24)$$

$$S = \frac{(79)(24)}{2} = 948$$

Recap

- Explicit definition of an arithmetic sequence $a_n = a_0 + nd$
- Find the sum of an arithmetic sequence by writing the sum forward and backward and adding vertically.