Solving 3 x 3 Systems of Linear Equations

Preliminaries and Objectives

Preliminaries
- Solving a 2 x 2 system of linear equations
  - Substitution Method
  - Elimination Method

Objectives
- Find the solution to a system of 3 equations in three variables.

Example 1

\[
\begin{align*}
(I) & \quad x + y + z = 0 \\
(II) & \quad -2x + 2y - 4z = 12 \\
(III) & \quad 2x - 3y - z = 7
\end{align*}
\]

\[
\begin{align*}
2 \cdot (I) \quad 2x + 2y + 2z &= 0 \\
(II) \quad -2x + 2y - 4z &= 12 \\
(IV) \quad 4y - 2z &= 12 \\
(V) \quad -y - 5z &= 19
\end{align*}
\]

\[
\begin{align*}
(V) \quad y + 20 &= 19 \\
x &= 3 \\
(3, 1, -4)
\end{align*}
\]

Example 2

\[
\begin{align*}
(I) & \quad -3x + 2y - 4z = 12 \\
(II) & \quad y + 2z = 5 \\
(III) & \quad 2x + 4y + 5z = 2
\end{align*}
\]

\[
\begin{align*}
2 \cdot (I) \quad -6x + 4y - 8z &= 24 \\
3 \cdot (I) \quad 6x + 12y + 15z &= 6 \\
(IV) \quad 16y + 7z &= 30
\end{align*}
\]

\[
\begin{align*}
-16 \cdot (II) \quad -16y - 32z &= -80 \\
25z &= -50 \\
z &= 2 \\
y &= 1 \\
x &= -6
\end{align*}
\]

\[(-6, 1, 2)\]

Recap

- Eliminate one variable using one pair of equations.
- Eliminate the same variable from a different pair of equations.
- Solve the resulting 2 x 2 system of equations.
- Substitute to find the values of the other variables.

Geometry of Linear Equations in 3-dimensional space

- Each linear equation is the equation of a plane.
- Two planes intersect in a line (usually).
- A line intersects the third equation (plane) at a point (usually).
- A false statement means two planes were parallel and there are no points of intersection of all three planes.
- A true statement means that there are infinitely many solutions, either because two planes were the same plane or that every pair of planes intersects at the same line.