Algebra
Activity 6c - Susceptible, Infected, Recovered (S-I-R) model

This activity explores various models of the spread of infectious diseases.

Part I - Exponential Growth

1. Suppose a single celled organism is allowed to grow freely, for example a bacteria in an apple. After one hour, the bacteria grows and divides in two. After two hours, the two bacteria subdivide into four. After three hours, there are eight bacteria.
   (a) Find the next three terms in the sequence $\{1, 2, 4, 8, \_\_\_, \_\_\_, \_\_\_, \ldots\}$
   (b) How many hours will it take for the number of bacteria to exceed 1000?
   (c) How many hours will it take for the number of bacteria to exceed 10 billion?
2. Now suppose that an individual infected with a virus spreads the disease to five others in a week.
   (a) Find the next three terms in the sequence $\{1, 5, 25, 125, \_\_\_, \_\_\_, \_\_\_, \ldots\}$
   (b) How many weeks will it take to infect the population of Plymouth MN (78,395)?
   (c) How many weeks will it take to infect the population of Hennepin County (1,252,000)?
   (d) How many weeks will it take to infect the population of Minnesota (5,700,000)?
   (e) How many weeks will it take to infect the population of the United States (332,000,000)?
3. Now suppose that an individual infected with a virus spreads the disease to only 0.8 others in a week and we begin with 100 infected people.
   (a) Find the next three terms in the sequence $\{100, 80, 64, 51.2, \_\_\_, \_\_\_, \_\_\_, \ldots\}$
   (b) How many weeks will it take until the number of infected people for the week is less than 1?
   (c) Make an estimate as to the number of total people infected by adding up all the terms in the sequence? (Note: The sequence is infinite, so you can't add all of the terms, but eventually the numbers are so small that they won't affect the sum much.)

Part II - S-I-R Model

4. A better model is the Susceptible - Infected - Recovered (S-I-R) model. The exponential model works well at first, but eventually, there are no more people to infect. The S-I-R model is based on the following values:

   - There is a 'spread rate' r, which is the number of people that an infected person comes in contact with in a week.
   - Let I_n be the number of infected individuals during week n.
   - Let S_n be the number of individuals that have never been infected and are therefore susceptible to catching the disease during week n.
   - Let R_n be the number of individuals that have been infected then recovered by week n.
   - Let P be the total population, so that $P = I_n + S_n + R_n$

   We will make the following assumptions:

   - An infected individual is infected for only one week, after which they recover.
   - Once recovered, an individual is immune, and won't become infected again.

   The recursions for the three values are as follows:

   - $R_{n+1} = R_n + I_n$. What this says is individuals recovered by time n stay recovered, and individuals infected at time n, recover at time $n+1$.
   - $I_{n+1} = (r)(I_n)\left(\dfrac{S_n}{P}\right)$. What this says is that each infected individual comes in contact with r individuals in the week, but only some of them are susceptible to the disease. You can't catch the disease if you are already infected or if you are recovered. $\dfrac{S}{P}$ is the percent chance that a contact actually results in a new infection.
   - $S_{n+1} = S_n - I_{n+1}$. What this says is the number of people that might catch the disease in the future are those that are currently susceptible minus those that catch the disease this week.

   In the following examples, if the number of infected individuals in any given week contains a decimal, we will round down.

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Example 1:

Let $r = 2.5$, $P = 1000$, $I_0 = 20$, $S_0 = 980$, $R_0 = 0$. This says that initially, there are 20 people infected. The other 980 people are susceptible, which represents 98% of the population. Each infected individual contacts 2.5 people.

In the first week, the number of new infections is $I_1 = (2.5)(20)(.98) = 49$. The 20 people previously infected then recover, so $R_1 = 20$. Of the 980 susceptible people, 49 became infected so $S_1 = 980 - 49 = 931$.

In the second week, the number of new infections is $I_2 = (2.5)(49)(.931) = 114$. The 49 people infected in week 1 then recover, so they are added to the 20 individuals previously recovered to get the total recovered $R_2 = 49 + 20 = 69$. Of the 931 susceptible people, 114 became infected so $S_2 = 931 - 114 = 817$.

Time	I_n	S_n	R_n
0	20	980	0
1	49	931	20
2	114	817	69
3			
4			
5			
6			
7			
8			

Continue filling in each row until the infection stops.

Example 2:

Let $r = 0,8$, $P = 1000$, $I_0 = 20$, $S_0 = 980$, $R_0 = 0$. This is the same as Example 1, except that now, the spread rate is only 0.8

Time	I_n	S_n	R_n
0	20	980	0
1	16	964	20
2			
3			
4			
5			
6			
7			
8			

Continue filling in each row until the infection stops.