Algebra
Activity 6a - Compounding Interest

The calculation of interest payments to a bank account is based on a percentage calculation at the end of one year. The percent is often called the Annual Percentage Rate (APR) and is a simple percent calculation. For example, if $\$5,000$ is deposited into a saving account and earns 1.25% interest annually, then at the end of one year, the amount of interest is $(5000)(0.0125) = 62.50$. In other words \$62.50 worth of interest is earned, making the current balance of the account \$5062.50. Note that the interest rate r is typically expressed as a percent, such as $r = 1.25\%$, but the number you should type on your calculator should be written 0.0125.

Interest can be calculated for more than one year. The balance to begin the next year is the balance at the end of the previous year. In the above example, we start year 2 with a balance of \$5062.50, which is again invested at a rate of 1.25%. The interest for the second year is $(5062.50)(0.0125) = 63.28$. In other words \$63.28 worth of interest is earned in the second year, making the balance of the account $\$5062.50 + \$63.28 = \$5125.78$ at the end of the second year.

Date	Beginning Balance	Interest ($r = 1.25\%$)	Ending Balance
Jan. 1, 2016	$5,000.00	$62.50	$5,062.50
Jan. 1, 2017	$5,062.50	$63.28	$5,125.78
Jan. 1, 2018	$5,125.78		
Jan. 1, 2019			
Jan. 1, 2020			

The goal of this activity is to produce a formula to calculate the accrued amount A (that is, A is the value of the investment at the end of the investment period), directly from the initial amount invested, called the principal P, the annual interest rate r, and the amount of time t that the money is invested, measured in years.

1. Complete the table above.

2. Suppose we invest \$3200 at 2.20% interest annually. For each of the four years from 2016 to 2020, calculate the interest each year and the balance at the end of each year to arrive at the total value of the investment. Note that the beginning balance for each year is the ending balance from the previous year.

Date	Beginning Balance	Interest ($r = 2.20\%$)	Ending Balance
Jan. 1, 2016	\$3200.00		
Jan. 1, 2017			
Jan. 1, 2018			
Jan. 1, 2019			
Jan. 1, 2020			

If it helps to have further examples, repeat the previous procedure for the following:

(a) $P = \$999, r = 19.99\%$

(b) $P = \$10000, r = 1\%$

(c) $P = \$750, r = 0.2\%$

3. Now let's generalize the previous problem. The investment principal is P and the interest rate is r.

   (a) Express the interest and ending balance as functions of P and r.

Date	Beginning Balance	Interest	Ending Balance
Jan. 1, 2016	P		

   (b) Your formula for the ending balance may involve P in more than one place. Factor out the P, so that the ending balance contains the variable P only once. You should have a way to calculate the ending balance directly from the starting balance in one step.

   (c) Describe a recursive process to calculate the ending balance based on the beginning balance. This process should remind you of a geometric sequence. Let P_t be the balance at the beginning of year t, with the initial principal $P = P_0$. Express P_1 in terms of P_0. Express P_{t+1} in terms of P_t. If it is useful, you may fill in values in the table below.

Date	Beginning Balance	Interest	Ending Balance
Jan. 1, 2016	P		
Jan. 1, 2017			
Jan. 1, 2018			
Jan. 1, 2019			
Jan. 1, 2020			

   (d) Write an expression for the value of the investment $A = P_4$ after four years, beginning with an initial value of P, invested at an interest rate of r. If it is useful, you may fill in values in the table below.

   (e) Write an expression for the value of the investment $A = P_t$ after t years, beginning with an initial value of P, invested at an interest rate of r.

4. Interest may be calculated more often than once per year. The process is the same, and the interest rate r is still expressed as an annual interest rate, but you will only receive a portion of annual interest each time period. For example, if we invest \$1800 at an annual interest rate of 2.40% compounded quarterly, how much will our investment be worth after two years? Below is a chart. Note that in the interest calculation for the first quarter of the year, we do not receive $(1800)(0.024) = 43.20$. Instead, we receive one quarter of that amount since the annual interest needs to be divided into four payments. However, after the first quarter, we will be earning interest on the interest, so that after one year, we will have earned more than \$43.20 in interest. The interest for the first quarter should be $(1800)(0.024)(\frac{1}{4}) = 10.80$.

   (a) Complete the following table assuming an initial investment of $P = \$1800$ and an annual interest rate of $r = .024$, compounded quarterly (that is, the interest is calculated $n = 4$ times per year). Some of the values have been provided.

Date	Beginning Balance	Interest	Ending Balance
Jan. 1, 2016	\$1800.00	\$10.80	
Apr. 1, 2016			
July 1, 2016			\$1832.59
Oct. 1, 2016			
Jan. 1, 2017			
Apr. 1, 2017			
July 1, 2017			
Oct. 1, 2017			
Jan. 1, 2018			

(b) Write an expression for the value of the investment A after two years, beginning with an initial value of $P = \$1,800$, invested at an interest rate of $r = 2.40\%$, compounded $n = 4$ times per year. This should be similar to your expression in question 3. What is the exponent counting?

(c) Write an expression for the value of the investment A after two years, beginning with an initial value of P, invested at an interest rate of r, compounded $n = 4$ times per year.

(d) Write an expression for the value of the investment A after two years, beginning with an initial value of P, invested at an interest rate of r, compounded n times per year.

(e) Write an expression for the value of the investment A after t years, beginning with an initial value of P, invested at an interest rate of r, compounded n times per year. This is the general formula for compounding interest.

This formula (perhaps in a slightly altered form) can be found on many websites including

`http://www.moneychimp.com/articles/finworks/fmfutval.htm`

Here are some calculators to verify your answer to question 4a. The 'annual addition' is \$0, since we are only depositing the principal at the beginning, and not making other periodic deposits.

`http://www.moneychimp.com/calculator/compound_interest_calculator.htm`
`https://www.calculatestuff.com/financial/compound-interest-calculator`
`https://www.bankrate.com/calculators/savings/compound-savings-calculator-tool.aspx`
`https://www.nerdwallet.com/banking/calculator/compound-interest-calculator`