Algebra
Activity 5e - The Quadratic Formula, Three Different Ways

This activity asks you to show that the roots of the parabola in general form $y = ax^2 + bx + c$ are given by the formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In most text books, this is stated as:

The solutions to the equation $ax^2 + bx + c = 0$ are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. **Verfying the formula by substitution**

   (a) Beginning with the root form of a parabola $y = a(x - r)(x - s)$ where r and s are the roots, expand to express the parabola in general form $y = ax^2 + bx + c$. Express the coefficients a, b and c in terms of a, r and s.

   $a =$ $b =$ $c =$

   (b) Verify that the roots r and s are given by the quadratic formula

   $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   by substituting the expressions found above and simplifying. When you have fully simplified the expression, you should get the answers r and s

   Hint: An intermediate step in the simplification will be

   $$\frac{s + r \pm \sqrt{(s - r)^2}}{2}$$

2. **Completing the square**

   Complete the square to solve the equation $ax^2 + bx + c = 0$

   There are several approaches which can be found online. One such approach is to perform the following steps:

   - Move the constant c to the right side.
   - Divide both sides by a.
   - Supply the missing constant to be able to factor $x^2 + \frac{b}{a}x + \ldots$ as a perfect square. Make sure you add this constant to both sides.
   - Simplify the right side by finding a common denominator.
   - Factor the left side as a perfect square.
   - Take a square root on both sides.
   - Solve for x

3. **Quadratic Formula from Standard Form**

(a) Solve the equation

$$a(x-h)^2 + k = 0$$

(b) Expand the standard form $y = a(x-h)^2 + k$ to express the parabola in general form $y = ax^2 + bx + c$. Express the coefficients a, b and c as functions of a, h and k.

$a =$ $\qquad$ $b =$ $\qquad$ $c =$

(c) Solve the above set of equations for a, h and k as functions of a, b and c.

$a =$ $\qquad$ $h =$ $\qquad$ $k =$

(d) Substitute the expressions in part c) into the solution from part a) and simplify.