Algebra
Activity 5c - Parabolas: Root Form

Another important feature of many functions is where the function crosses the x-axis (the x-intercepts). Note that x-intercepts are also called **roots** of the function and also called **zeroes** of the function since they are solutions to the equation $f(x) = 0$. This activity makes connections between the roots and the other forms of a parabola (the graph, the standard form and the general form).

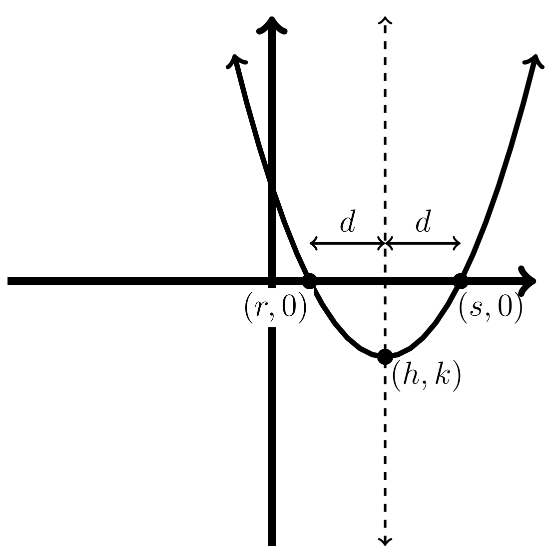
Part I - $a = 1$

Suppose $y = (x - r)(x - s)$; r and s are called roots of the equation.

1. Verify that if $x = r$, then $y = 0$ and that if $x = s$, then $y = 0$. This verifies that r and s are x-intercepts.

2. Expand the root form $y = (x - r)(x - s)$ to arrive at the general form $y = x^2 + bx + c$. Express b and c in terms of r and s.

3. Knowing a formula for h in terms of b, namely $h = -\frac{b}{2a}$, which in this case becomes $h = -\frac{b}{2}$ since $a = 1$, express h in terms of r and s. Why does this make sense as a result of the symmetry of the graph of the parabola?



4. Let d be the distance from the axis of symmetry to the roots. Explain why the x-intercepts are a vertical distance of d^2 higher than the vertex. In terms of d, what is the value of k? Also express k as a function of r and s.

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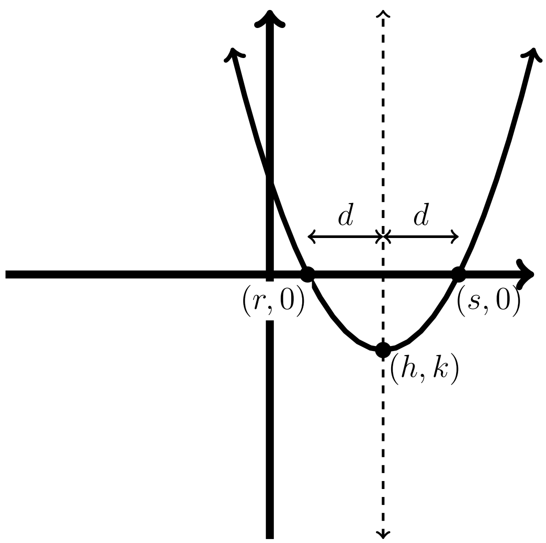
Part II - Arbitrary values of a

Suppose $y = a(x - r)(x - s)$

1. Expand the root form $y = a(x - r)(x - s)$ to arrive at the general form $y = ax^2 + bx + c$. Express a, b and c in terms of a, r and s.

2. Knowing a formula for h in terms of a and b, namely $h = -\frac{b}{2a}$, express h in terms of a, r and s.



3. Let d be the distance from the axis of symmetry to the roots. How far above the vertex are the x-intercepts? This is similar to question 4 of Part I, except this graph is stretched by a factor of a. Express k as a function of a and d.

4. Express d as a function of a and k.

5. From the picture, it should be clear that the two roots r and s are $h \pm d$. Express the roots in terms of a, h and k.

6. Knowing formulas for h and k in terms of a, b, and c, express the roots in terms of a, b and c.