MATH 1051 - Slopes and Parabolas

We begin by examining the slope between two points on the parabola \( y = x^2 \). Some of the points on the parabola are \((0, 0), (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)\)

1. Find the slope between ...
   (a) the point \((2, 4)\) and the point \((5, 25)\)
   (b) the point \((2, 4)\) and the point \((4, 16)\)
   (c) the point \((2, 4)\) and the point \((3, 9)\)
   (d) the point \((2, 4)\) and the point \((1, 1)\)

   Can you guess a simple formula to calculate the slope between \((2, 4)\) and an arbitrary point on the parabola?

   What is the slope between the point \((2, 4)\) and the point \((2, 4)\)?

2. Suppose the second point on the parabola is unknown, that is, suppose that the second point is \((x, x^2)\) and that we wish to find the slope between the point \((2, 4)\) and the point \((x, x^2)\).
   - Use the slope formula to write an expression for the slope between these two points.
   - Factor and cancel to simplify this formula.
   - Does this formula agree with what you guessed in part 1?

3. Find the slope between ...
   (a) the point \((3, 9)\) and the point \((5, 25)\)
   (b) the point \((3, 9)\) and the point \((4, 16)\)
   (c) the point \((3, 9)\) and the point \((2, 4)\)
   (d) the point \((3, 9)\) and the point \((1, 1)\)

   Can you guess a simple formula to calculate the slope between \((3, 9)\) and an arbitrary point on the parabola?

4. Find the slope between ...
   (a) the point \((4, 16)\) and the point \((5, 25)\)
   (b) the point \((4, 16)\) and the point \((3, 9)\)
   (c) the point \((4, 16)\) and the point \((2, 4)\)
   (d) the point \((4, 16)\) and the point \((1, 1)\)

   Can you guess a simple formula to calculate the slope between \((4, 16)\) and an arbitrary point on the parabola?

5. Can you guess what the slope is between any two points on a parabola?

6. Suppose both points on the parabola are unknown, that is, suppose that the second point is \((x, x^2)\) and that the first fixed point is \((h, h^2)\). We wish to find the slope between the point \((h, h^2)\) and the point \((x, x^2)\).
   - Use the slope formula to write an expression for the slope between these two points.
   - Factor and cancel to simplify this formula.
   - Does this formula agree with what you guessed in part 5?