MATH 1051 - Slopes and Parabolas

We begin by examining the slope between two points on the parabola $y = x^2$. Some of the points on the parabola are $(0, 0), (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)$

1. Find the slope between ...

   (a) the point $(2, 4)$ and the point $(5, 25)$
   (b) the point $(2, 4)$ and the point $(4, 16)$
   (c) the point $(2, 4)$ and the point $(3, 9)$
   (d) the point $(2, 4)$ and the point $(1, 1)$

   Can you guess a simple formula to calculate the slope between $(2, 4)$ and an arbitrary point on the parabola?

   What is the slope between the point $(2, 4)$ and the point $(2, 4)$?

2. Suppose the second point on the parabola is unknown, that is, suppose that the second point is (x, x^2) and that we wish to find the slope between the point $(2, 4)$ and the point (x, x^2).

   - Use the slope formula to write an expression for the slope between these two points.
   - Factor and cancel to simplify this formula.
   - Does this formula agree with what you guessed in part 1?

3. Find the slope between ...

   (a) the point $(3, 9)$ and the point $(5, 25)$
   (b) the point $(3, 9)$ and the point $(4, 16)$
   (c) the point $(3, 9)$ and the point $(2, 4)$
   (d) the point $(3, 9)$ and the point $(1, 1)$

   Can you guess a simple formula to calculate the slope between $(3, 9)$ and an arbitrary point on the parabola?

4. Find the slope between ...

   (a) the point $(4, 16)$ and the point $(5, 25)$
   (b) the point $(4, 16)$ and the point $(3, 9)$
   (c) the point $(4, 16)$ and the point $(2, 4)$
   (d) the point $(4, 16)$ and the point $(1, 1)$

   Can you guess a simple formula to calculate the slope between $(4, 16)$ and an arbitrary point on the parabola?

5. Can you guess what the slope is between any two points on a parabola?

6. Suppose both points on the parabola are unknown, that is, suppose that the second point is (x, x^2) and that the first fixed point is (h, h^2). We wish to find the slope between the point (h, h^2) and the point (x, x^2).

   - Use the slope formula to write an expression for the slope between these two points.
   - Factor and cancel to simplify this formula.
   - Does this formula agree with what you guessed in part 5?