Part I

1. Multiply \( \frac{21}{23} \times 23 \)

2. Multiply \((20 + 1)(20 + 3)\)
   
   Explain why problems 1) and 2) are the same process.

   If you learned other ways to multiply two-digit numbers, show them to your group and explain why it is the same process as problems 1) and 2)

3. Multiply \((2x + 1)(2x + 3)\)
   
   Explain why this is the same process as problems 1) and 2)
Activity 1a - Multiplication via the FOIL Method and Factoring

Part II

Here is the product \((2x + 1)(2x + 3)\) performed by the ‘Box Method’. The factors are placed outside the grid, and are split into the components that are added and subtracted (i.e., the ‘terms’). Inside the grid are placed the products of these individual terms, which are then added to get the final product.

\[
\begin{array}{cc}
2x & +1 \\
2x & \\
+3 & \\
\end{array}
\quad \Rightarrow \quad
\begin{array}{ccc}
2x & 4x^2 & 2x \\
+3 & 6x & 3 \\
\end{array}
\quad \Rightarrow \quad (2x + 1)(2x + 3) = 4x^2 + 8x + 3
\]

We now wish to undo the multiplication process by returning the product to its original factors. Here is an example:

\[
\begin{array}{cc}
x & +5 \\
x^2 & 5x \\
3x & 15 \\
\end{array}
\quad \Rightarrow \quad
\begin{array}{ccc}
x & x^2 & 5x \\
+3 & 3x & 15 \\
\end{array}
\quad \Rightarrow \quad x^2 + 8x + 15 = (x + 5)(x + 3)
\]

When the four parts of the FOIL process are separated in the grid, the factors can be determined by finding the common factor in each row and column.

Factor the following:

1. 
\[x^2 - 7x + 10\]

\[
\begin{array}{cc}
x^2 & -5x \\
-2x & 10 \\
\end{array}
\]

2. 
\[x^3 - x^2 + 3x - 3\]

\[
\begin{array}{cc}
x^3 & -x^2 \\
3x & -3 \\
\end{array}
\]
3. 

\[ x^3 - 4x^2 + 5x - 20 \]

4. 

\[ x^2 + 3x - 28 \]

5. 

\[ x^2 + 6x + 8 \]

6. 

\[ x^2 - 9 \]

7. List all of the possible values of \( b \) for the factorization of \( x^2 + bx - 12 \).
Part III

Expand the following:

1. \((x + 1)(x + 1)\)

2. \((x + 1)^2\)

3. \((x + 1)(x + 1)(x + 1)\)

4. \((x + 1)^3\)

5. \((x + 1)^4\)

6. \((x + 1)^5\)

Do you see a pattern?