

The Inverse Functions of Sine and Cosine



Preliminaries and Objectives

Preliminaries:

- Values of $\sin x$ and $\cos x$ from unit circle in both radians and degrees.

Objectives:

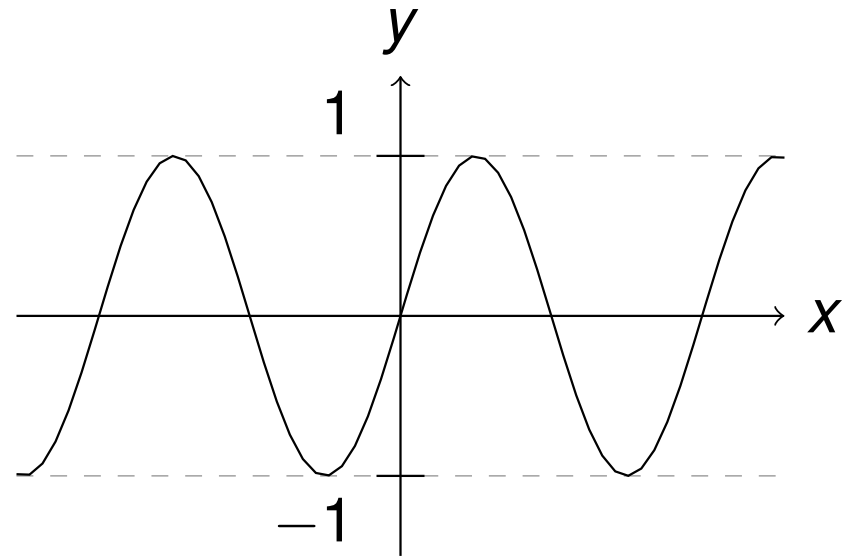
- Find values of the inverse sine and inverse cosine functions.

The sin Function

$$y = \sin x$$

x is an angle

$$-1 \leq y \leq 1$$



The sin Function

$$y = \sin x$$

x is an angle

$$-1 \leq y \leq 1$$

$$y = \sin^{-1} x$$

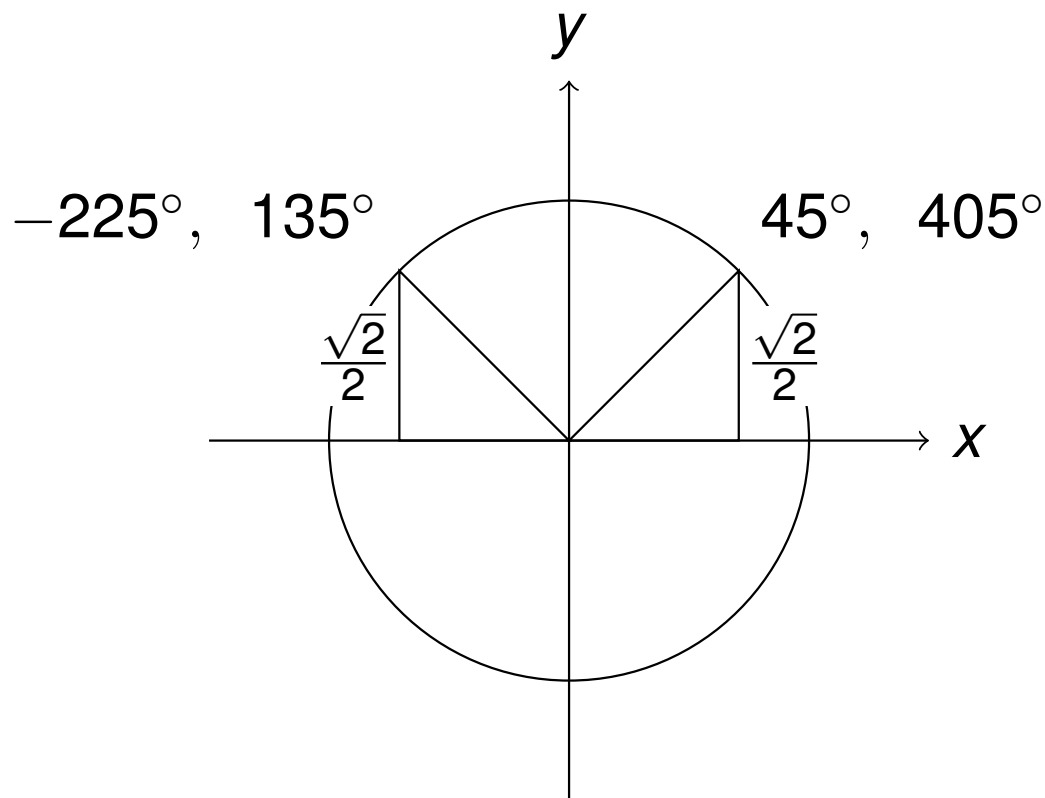
$$x = \sin y$$

y is the angle
whose sin is x

$$-1 \leq x \leq 1$$

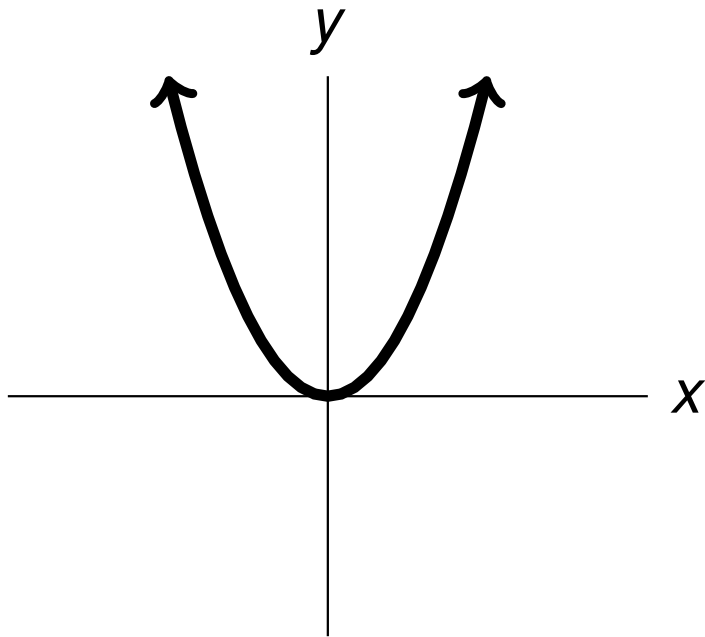
The Inverse Sine Function

$$y = \sin^{-1} \left(\frac{\sqrt{2}}{2} \right)$$



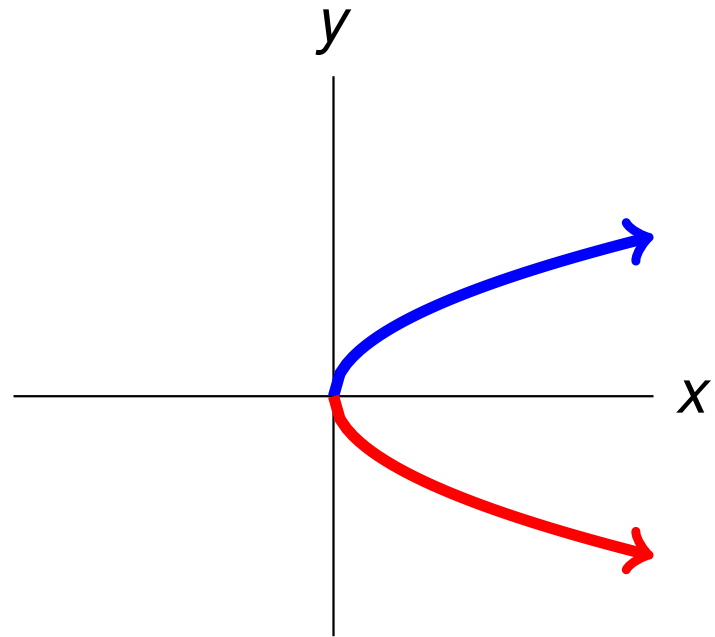
The Inverse of $y = x^2$

$$y = x^2$$



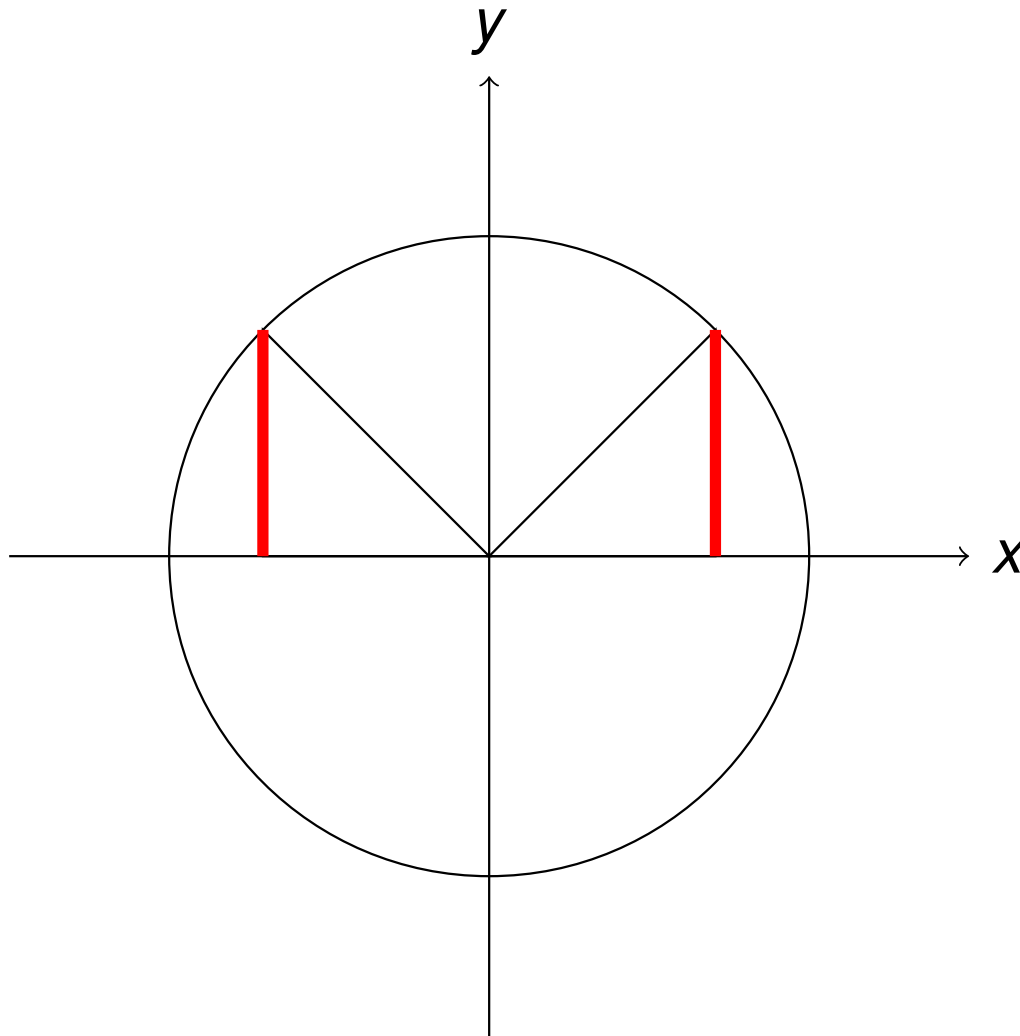
$$x = y^2$$

$$y = \pm\sqrt{x}$$



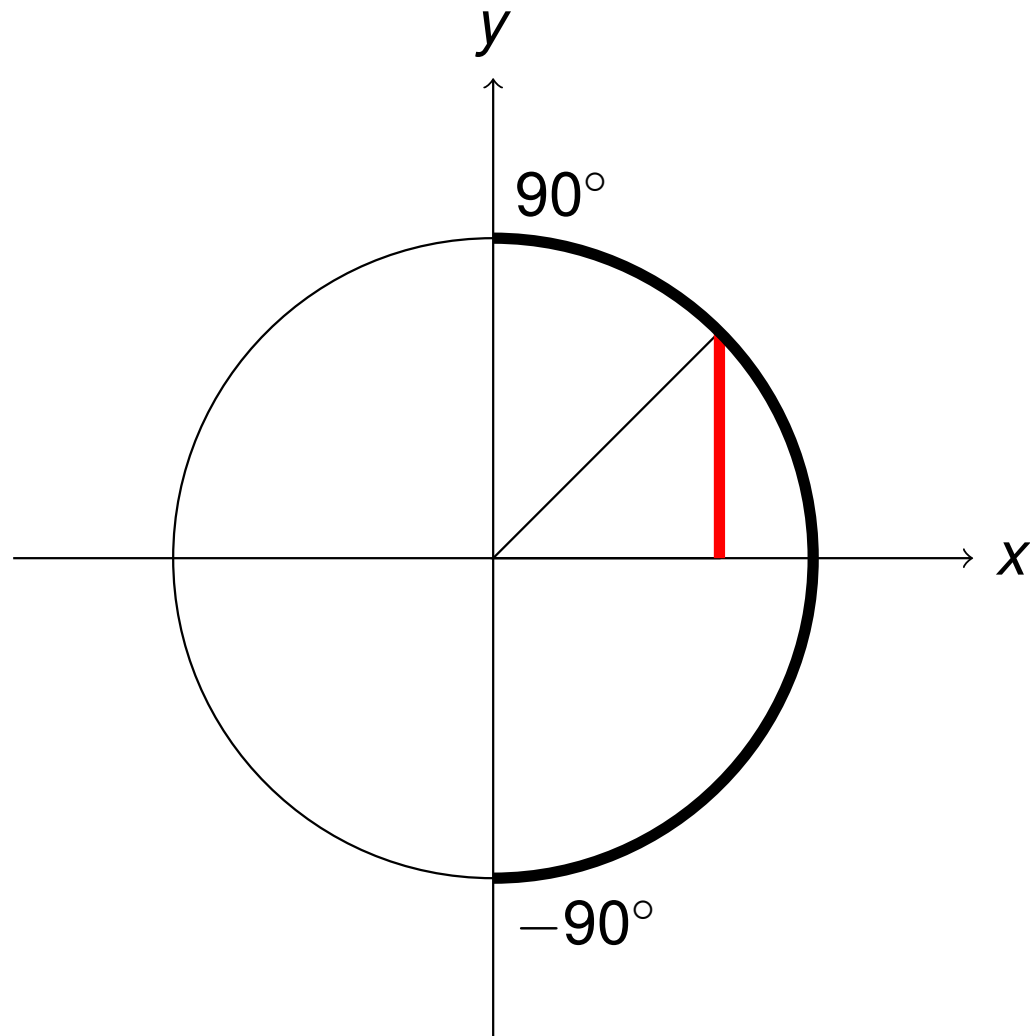
The Inverse Sine Function

$$\sin^{-1} \frac{\sqrt{2}}{2}$$



The Inverse Sine Function

$$\sin^{-1} \frac{\sqrt{2}}{2} = 45^\circ$$



The Inverse Sine Function

$$\sin^{-1}(-1) = -90^\circ$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ$$

$$\sin^{-1}(0) = 0^\circ$$

$$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\sin^{-1}(1) = 90^\circ$$

The Inverse Sine Function

$$\sin^{-1}(-1) = -\frac{\pi}{2}$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\sin^{-1}(0) = 0$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

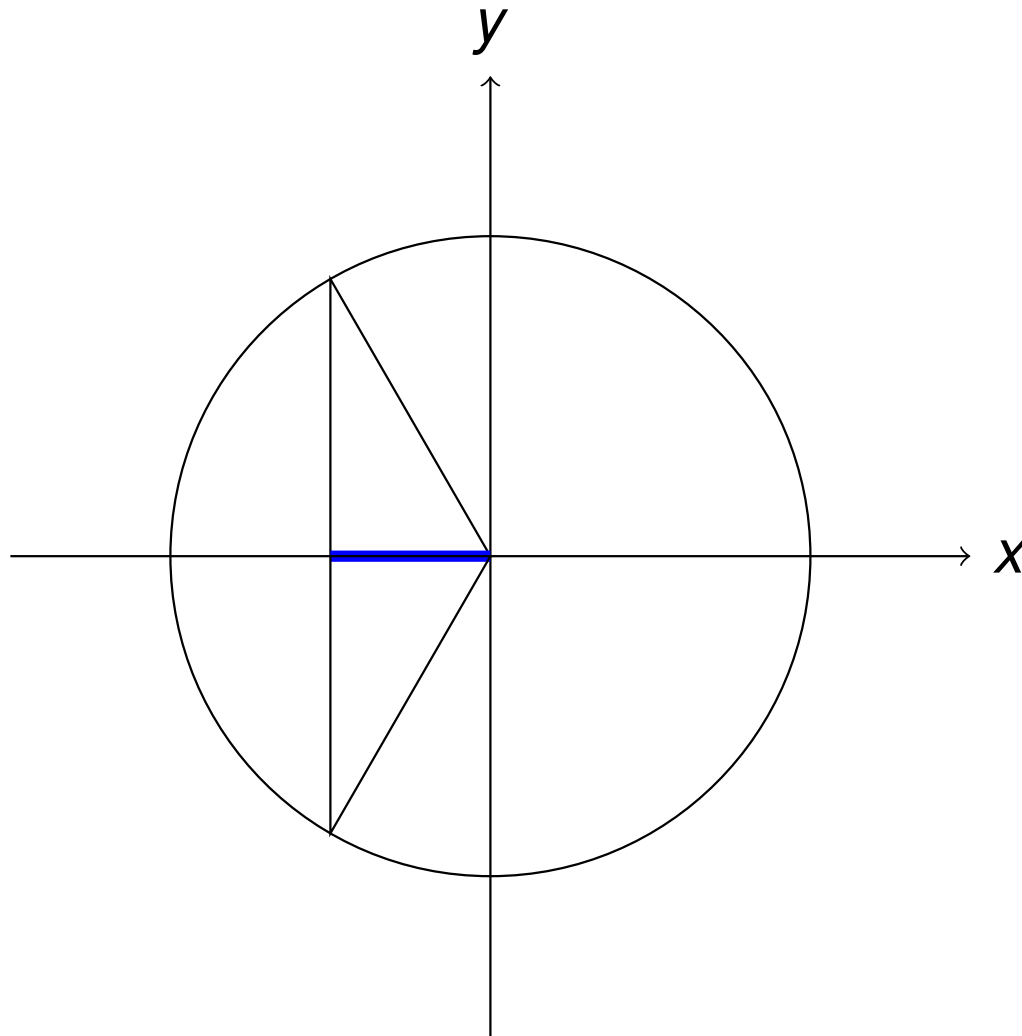
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$\sin^{-1}(1) = \frac{\pi}{2}$$

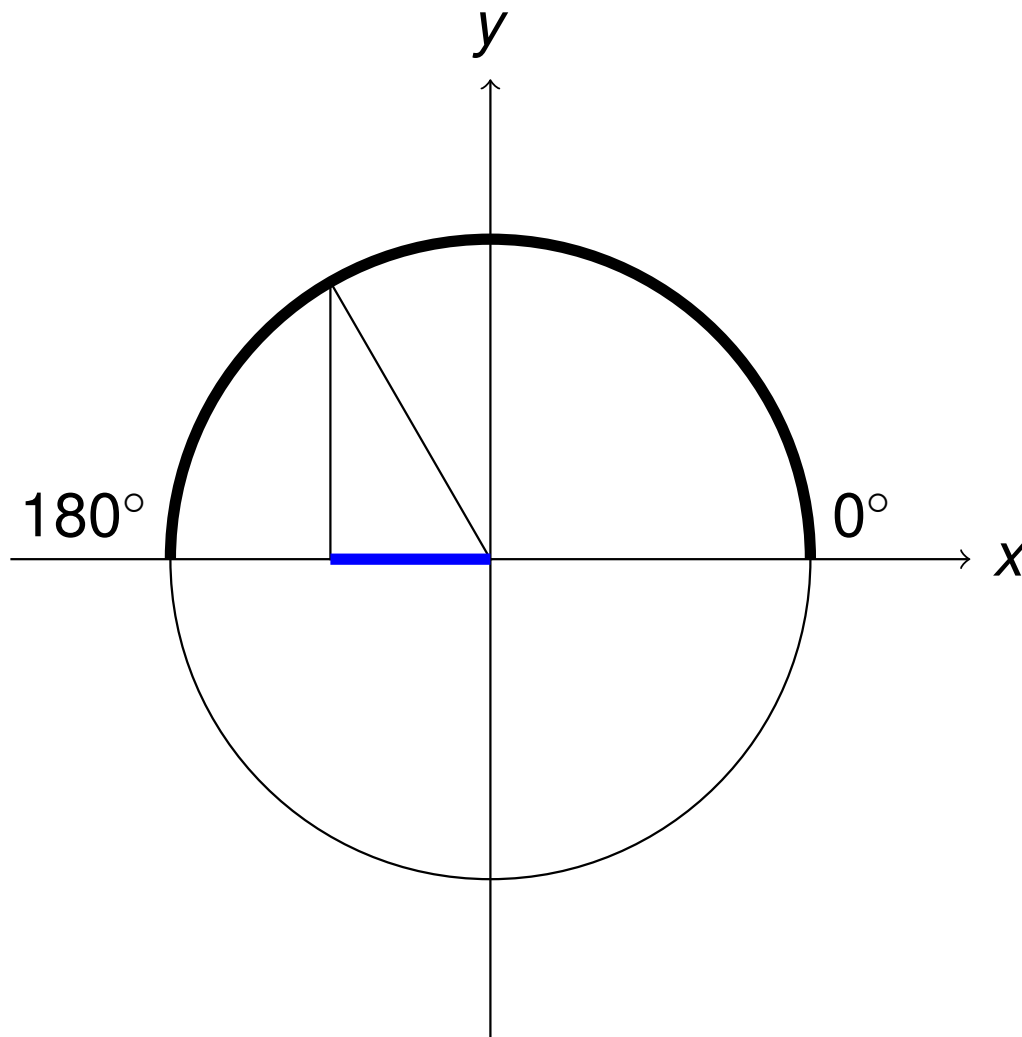
The Inverse Cosine Function

$$\cos^{-1} -\frac{1}{2}$$



The Inverse Cosine Function

$$\cos^{-1} -\frac{1}{2} = 120^\circ$$



The Inverse Cosine Function

$$\cos^{-1}(-1) = 180^\circ$$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135^\circ$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$$

$$\cos^{-1}(0) = 90^\circ$$

$$\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

$$\cos^{-1}(1) = 0^\circ$$

The Inverse Cosine Function

$$\cos^{-1}(-1) = \pi$$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\cos^{-1}(0) = \frac{\pi}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\cos^{-1}(1) = 0$$

Recap

- For $y = \sin^{-1} x$ and $y = \cos^{-1} x$, x is a number between -1 and 1 and y is an angle.
- For $y = \sin^{-1} x$, the answer must be between -90° and 90° .
- For $y = \cos^{-1} x$, the answer must be between 0° and 180° .