General Equation of an Ellipse
Preliminaries and Objectives

Preliminaries
- Equation of a circle
- Transformation of graphs (shifting and stretching)

Objectives
- Find the equation of an ellipse, given the graph.
Circle centered at the origin

\[ x^2 + y^2 = r^2 \]

\[ \frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \]

\[ \left( \frac{x}{r} \right)^2 + \left( \frac{y}{r} \right)^2 = 1 \]
\[ y = \sin(Bx) \]

The sine wave is \( B \) times thinner. Period (wavelength) is divided by \( B \). Frequency is multiplied by \( B \).

\[ y = \sin\left(\frac{x}{b}\right) \]

The sine wave is \( b \) times wider. Period (wavelength) is multiplied by \( b \). Frequency is divided by \( b \).

\[
\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1
\]

The unit circle is stretched \( r \) times wider and \( r \) times taller.
Ellipse Centered at the Origin

\[
\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1
\]

The unit circle is stretched \( r \) times wider and \( r \) times taller.

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1
\]

The unit circle is stretched \( a \) times wider and \( b \) times taller.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
Ellipse centered at the origin

\[ \frac{x^2}{9} + \frac{y^2}{25} = 1 \]
Ellipse centered at the origin

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
\[
\frac{x^2}{16} + \frac{y^2}{36} = 1
\]
General Form of an Ellipse

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]

Center at \((h, k)\)

Vertices at \((h + a, k), (h - a, k), (h, k + b), (h, k - b)\)
Example 1

Graph \[ 9(x - 3)^2 + 16(y + 2)^2 = 144 \]

\[ \frac{(x - 3)^2}{16} + \frac{(y + 2)^2}{9} = 1 \]
Example 2

General Equation of an Ellipse

\[
\frac{(x + 2)^2}{16} + \frac{(y + 1)^2}{36} = 1
\]
Recap

General Equation of an Ellipse

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]

Center at \((h, k)\)

Vertices at \((h + a, k), (h - a, k), (h, k + b), (h, k - b)\)