Double and Half Angle Formulas

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries

- Be able to derive the double angle formulas from the angle sum formulas
- Inverse trig functions
- Simplify fractions
- Rationalize the denominator

Objectives

- Use the double angle formulas to find specific values

Double and Half Angle Formulas

$$\sin(2A) = 2\sin A\cos A$$

$$\begin{aligned}\cos(2A) &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

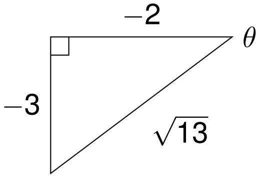
$$\begin{aligned}\tan\left(\frac{A}{2}\right) &= \frac{\sin A}{1+\cos A} \\ &= \frac{1-\cos A}{\sin A}\end{aligned}$$

Double and Half Angle Formulas

Find $\sin(2\theta)$, $\tan(2\theta)$ and $\tan(\frac{\theta}{2})$ if $\tan\theta = \frac{3}{2}$, where $\pi < \theta < \frac{3\pi}{2}$

Solution:

$$\sin\theta = \frac{-3}{\sqrt{13}},\ \cos\theta = \frac{-2}{\sqrt{13}},\ \tan\theta = \frac{3}{2}$$



$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta \\ &= 2\left(\frac{-3}{\sqrt{13}}\right)\left(\frac{-2}{\sqrt{13}}\right) \\ &= \frac{12}{13}\end{aligned}$$

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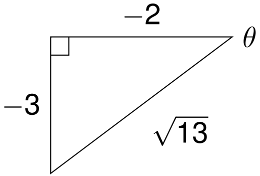

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2(\frac{3}{2})}{1-(\frac{3}{2})^2} = \frac{3}{-\frac{5}{4}} = -\frac{12}{5}$$

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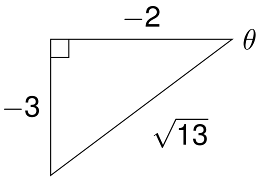


$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{-\frac{3}{\sqrt{13}}}{1-\frac{2}{\sqrt{13}}} = \frac{-2-\sqrt{13}}{3}$$

Previous Answer with slower algebra

$$\frac{-\frac{3}{\sqrt{13}}}{1-\frac{2}{\sqrt{13}}} = \frac{-\frac{3}{\sqrt{13}}\cdot\sqrt{13}}{\left(1-\frac{2}{\sqrt{13}}\right)\cdot\sqrt{13}} = \frac{-3}{\sqrt{13}-2}$$

$$= \frac{-3}{\left(\sqrt{13}-2\right)}\frac{\left(\sqrt{13}+2\right)}{\left(\sqrt{13}+2\right)} = \frac{-3\sqrt{13}-6}{13-4}$$

$$= \frac{3\left(-\sqrt{13}-2\right)}{9} = \frac{-2-\sqrt{13}}{3}$$

Double and Half Angle Formulas

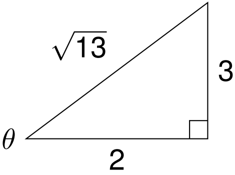
Find $\sin(2\tan^{-1}\frac{3}{2})$

Solution:

Let $\theta = \tan^{-1}\frac{3}{2}$ so that we are are asked to find $\sin(2\theta)$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin\theta = \frac{3}{\sqrt{13}},\ \cos\theta = \frac{2}{\sqrt{13}}$$



$$\Rightarrow \sin(2\theta) = 2\left(\frac{3}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right) = \frac{12}{13}$$

Recap

- Draw and label triangles for each given trig value
- Use Pythagorean Theorem to find missing lengths
- Write down the appropriate formula
- Plug in values of trig functions from the triangles in steps 1 and 2
- Simplify