Combining Trig Functions and Inverse Trig Functions - Part II

Preliminaries and Objectives

Preliminaries:

- Trig functions
- Inverse Trig Functions
- Angle Sum Formulas

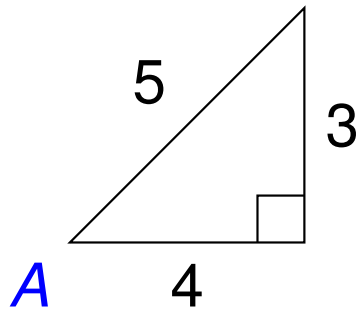
Objectives:

- Find values when trig functions and inverse trig functions are combined in angle sum formulas.

Angle Sum Formulas - Example 1

Find

$$\sin\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{2}{3}\right)\Leftrightarrow \sin\left(A+B\right)$$

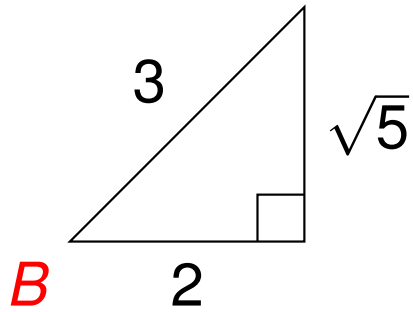


$$\tan A=\frac{3}{4}\ ;\ \sin A=\frac{3}{5}\ ;\ \cos A=\frac{4}{5}$$

$$\cos B=\frac{2}{3}\ ;\ \sin B=\frac{\sqrt{5}}{3}\ ;\ \tan B=\frac{\sqrt{5}}{2}$$

$$\sin\left(A+B\right)=(\sin A)(\cos B)+(\cos A)(\sin B)$$

$$=\left(\frac{3}{5}\right)\left(\frac{2}{3}\right)+\left(\frac{4}{5}\right)\left(\frac{\sqrt{5}}{3}\right)=\frac{6+4\sqrt{5}}{15}$$

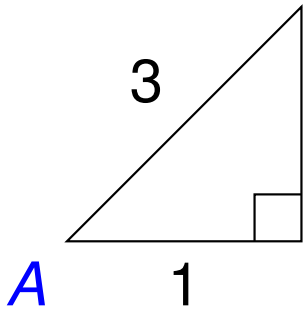


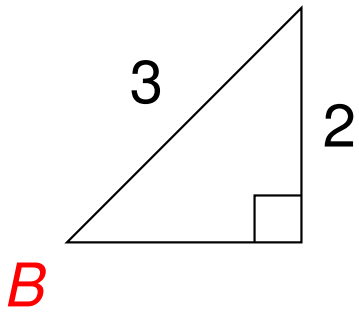
Angle Sum Formulas - Example 2

Find

$$\cos\left(\sec^{-1} 3 + \sin^{-1}\frac{2}{3}\right) \Leftrightarrow \cos\left(A + B\right)$$

$$\sec A = 3\ ;\ \cos A = \frac{1}{3}$$





Angle Sum Formulas - Example 3

To find

$$\csc\left(\tan^{-1}\tfrac{3}{4}+\cos^{-1}\tfrac{2}{3}\right)\Leftrightarrow \csc\left(A+B\right)\Leftrightarrow \frac{1}{\sin\left(A+B\right)}$$

first find

$$\sin\left(\tan^{-1}\tfrac{3}{4}+\cos^{-1}\tfrac{2}{3}\right)=\tfrac{6+4\sqrt{5}}{15}$$

then take the reciprocal to get

$$\csc\left(\tan^{-1}\tfrac{3}{4}+\cos^{-1}\tfrac{2}{3}\right)=\frac{15}{6+4\sqrt{5}}=\tfrac{30\sqrt{5}-45}{22}$$

Recap

- Inverse trig functions give information about two sides of a triangle
- The third side can be found by the Pythagorean Theorem
- Values of trig functions can be found from the triangle and plugged into the appropriate formula.

This technique will also work for double and half angle formulas.

Angle Sum Formulas - Solution to Example 2

In case you wanted the answer to the second example:

$$\cos\left(\sec^{-1} 3 + \sin^{-1}\tfrac{2}{3}\right) \Leftrightarrow \cos\left(A + B\right)$$

$$\sec A = 3\ ;\ \cos A = \tfrac{1}{3}\ ;\ \sin A = \tfrac{2\sqrt{2}}{3}$$

$$\sin B = \tfrac{2}{3}\ ;\ \cos B = \tfrac{\sqrt{5}}{3}$$

$$\cos\left(\sec^{-1} 3 + \sin^{-1}\tfrac{2}{3}\right) = \left(\tfrac{1}{3}\right)\left(\tfrac{\sqrt{5}}{3}\right) - \left(\tfrac{2\sqrt{2}}{3}\right)\left(\tfrac{2}{3}\right) = \tfrac{\sqrt{5}-4\sqrt{2}}{9}$$