Combining Trig Functions and Inverse Trig Functions - Part I

Preliminaries and Objectives

Preliminaries:

- Trig functions
- Inverse Trig Functions

Objectives:

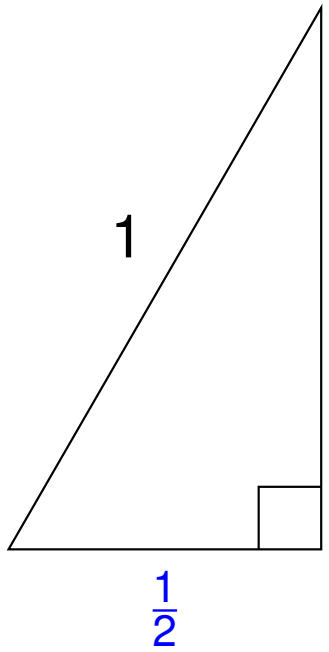
- Find values when trig functions and inverse trig functions are combined.

Example 1

Find

$$\sin\left(\cos^{-1}\frac{1}{2}\right)$$

$\frac{1}{2}$ → number

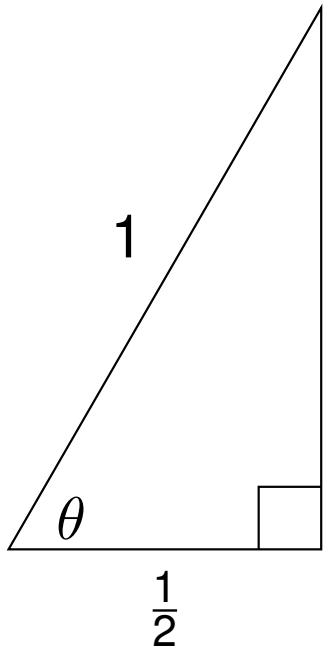


Example 1

Find

$$\sin\left(\cos^{-1}\frac{1}{2}\right)$$

angle

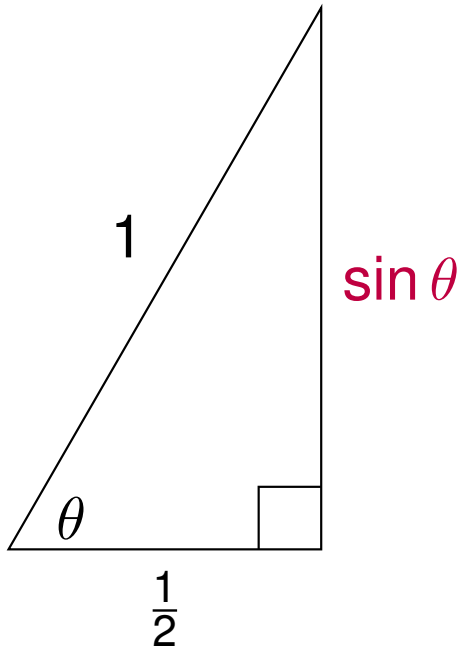


$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Example 1

Find

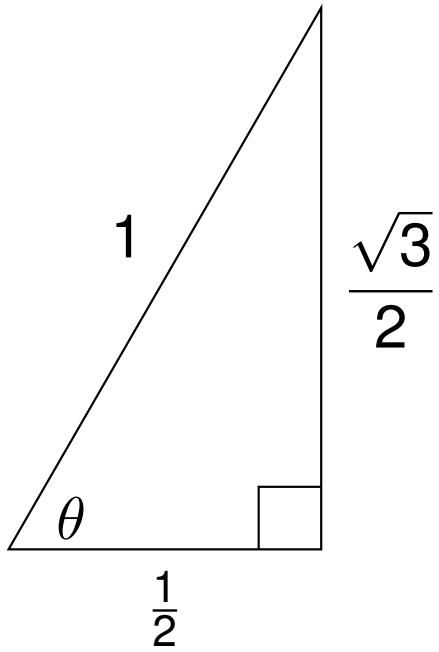
$$\underbrace{\sin\left(\cos^{-1}\frac{1}{2}\right)}_{\text{number}}$$



Example 1

Find

$$\sin\left(\cos^{-1}\frac{1}{2}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

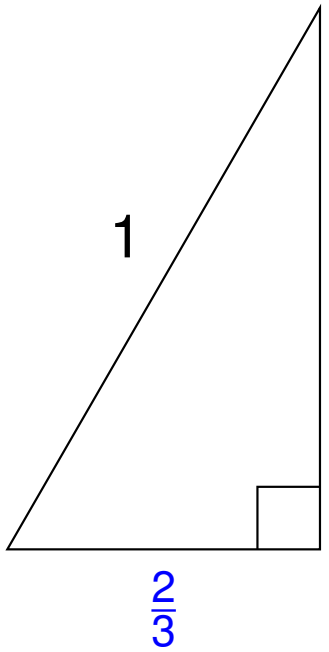


Example 2

Find

$$\sin\left(\cos^{-1}\frac{2}{3}\right)$$

number

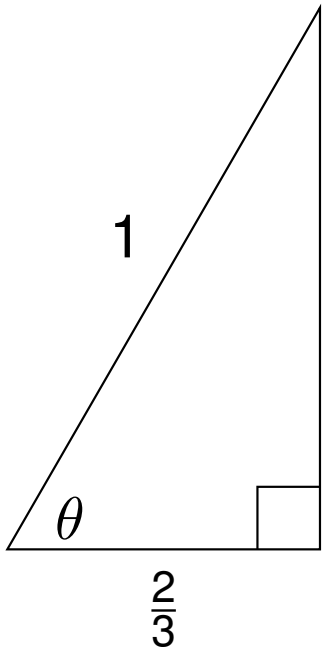


Example 2

Find

$$\sin\left(\cos^{-1}\frac{2}{3}\right)$$

angle



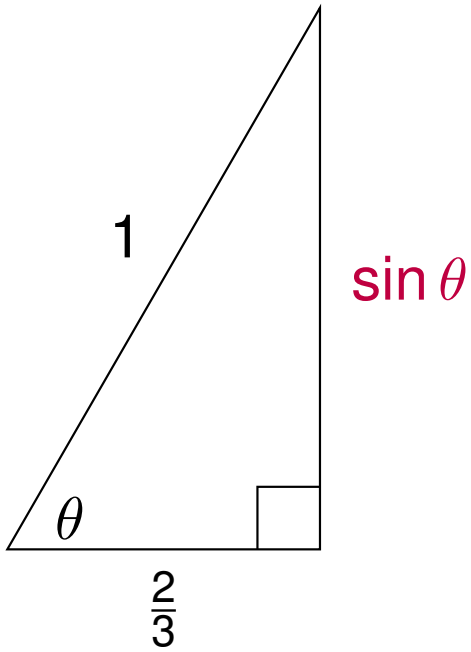
$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Example 2

Find

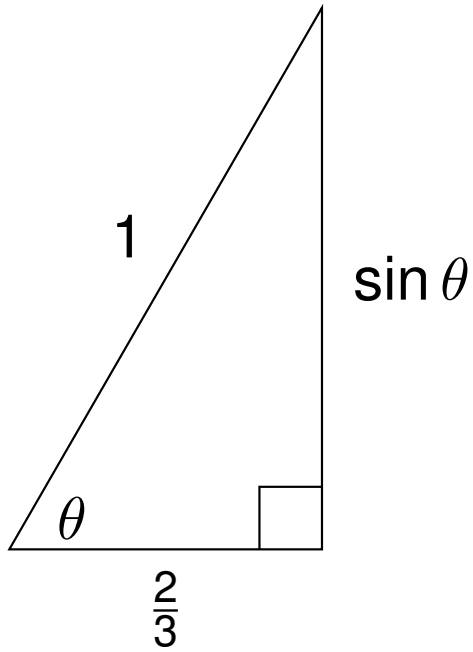
$$\sin\left(\cos^{-1}\frac{2}{3}\right)$$

number



Example 2

If $\cos\theta = \frac{2}{3}$ and θ is in quadrant I, find $\sin\theta$.



Example 2

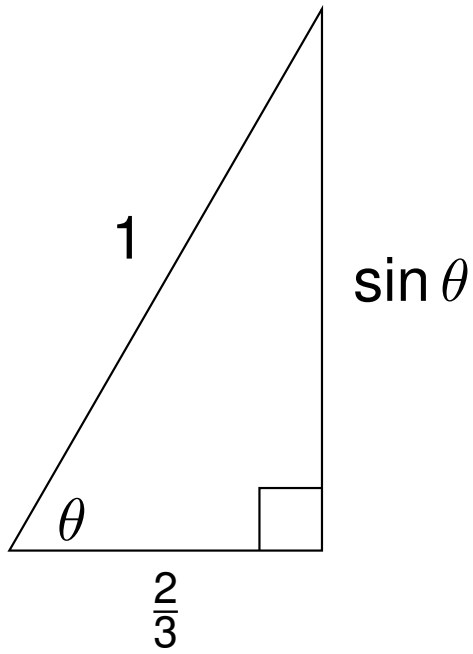
$$\left(\frac{2}{3}\right)^2 + (\sin\theta)^2 = 1$$

$$(\sin\theta)^2 = 1 - \frac{4}{9}$$

$$(\sin\theta)^2 = \frac{5}{9}$$

$$\sin\theta = \frac{\sqrt{5}}{3}$$

$$\sin\left(\cos^{-1}\frac{2}{3}\right) = \frac{\sqrt{5}}{3}$$



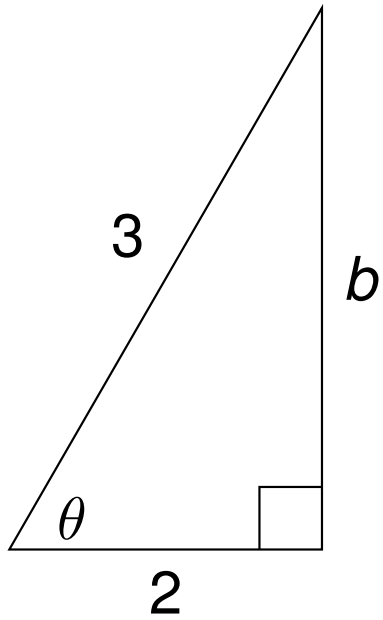
Example 2

$$\sin\left(\cos^{-1}\frac{2}{3}\right)$$

$$b^2 + 2^2 = 3^2$$

$$b^2 = 9 - 4 = 5$$

$$b = \pm\sqrt{5}$$



$$\sin\theta = \frac{b}{3} = \frac{\sqrt{5}}{3}$$

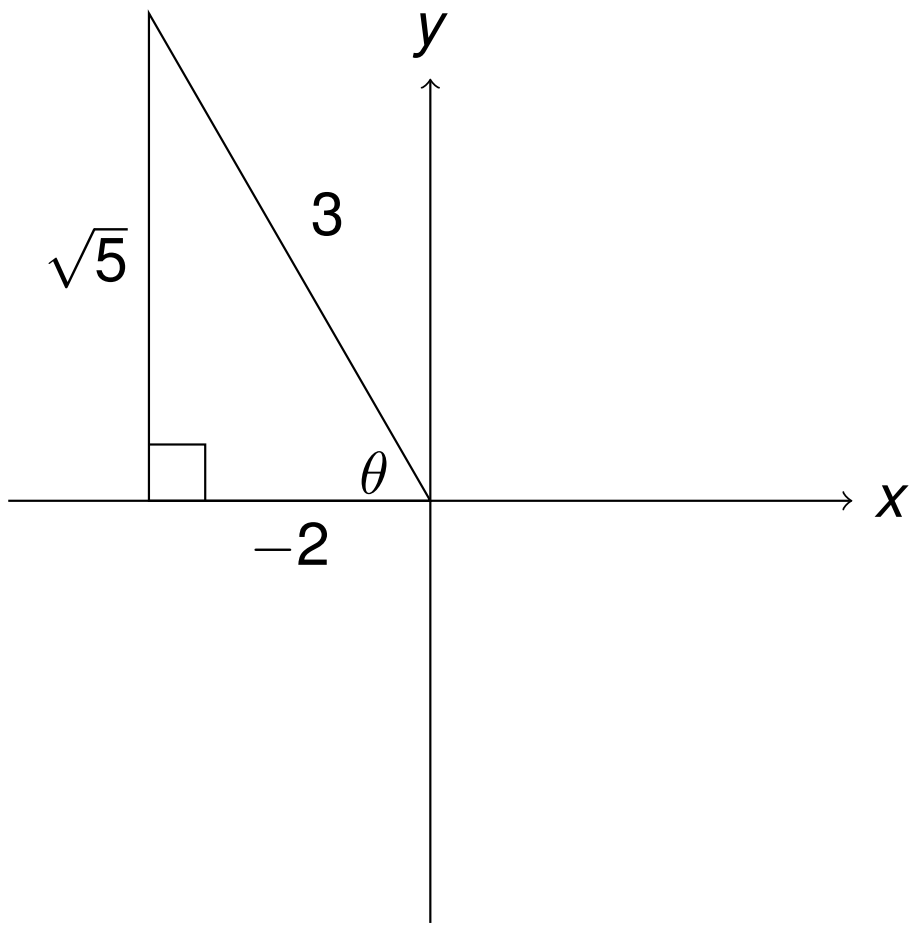
$$\sin\left(\cos^{-1}\frac{2}{3}\right) = \frac{\sqrt{5}}{3}$$

Example 3 - Negative Values

Find

$$\sin\left(\cos^{-1}\ -\frac{2}{3}\right)$$

$$= \sin\theta = \frac{\sqrt{5}}{3}$$

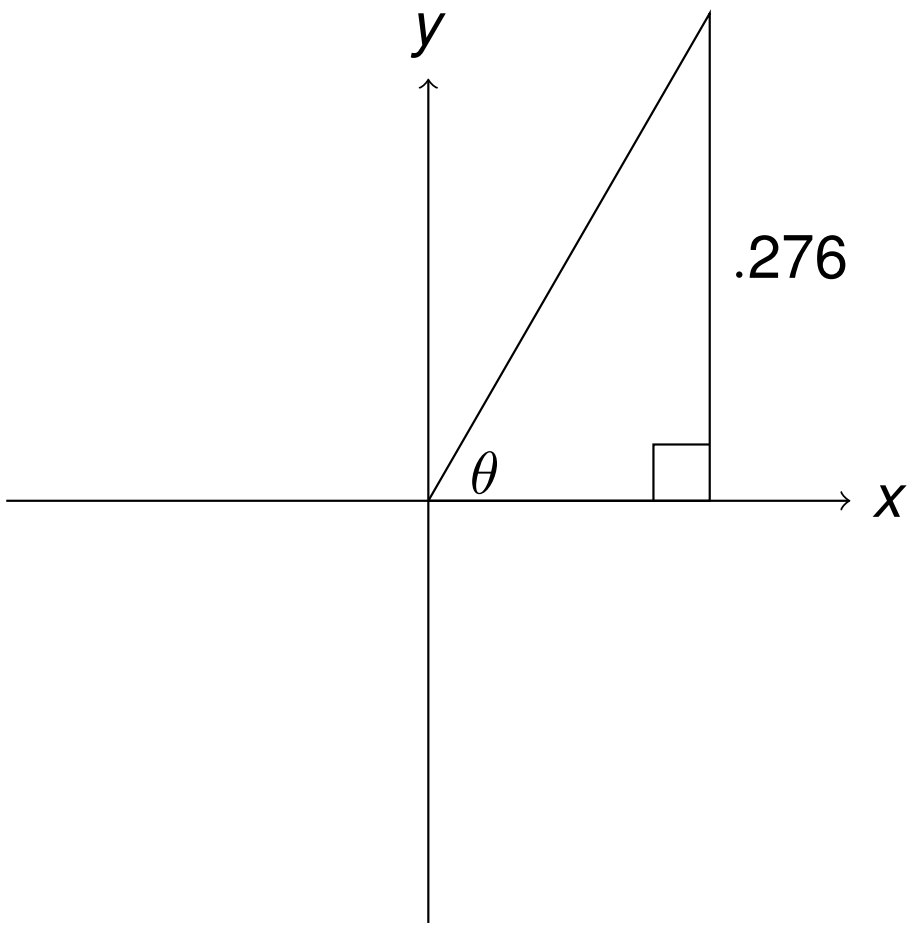


Example 4

Find

$$\sin\left(\sin^{-1}\ .276\right)$$

$$\sin\theta = .276$$



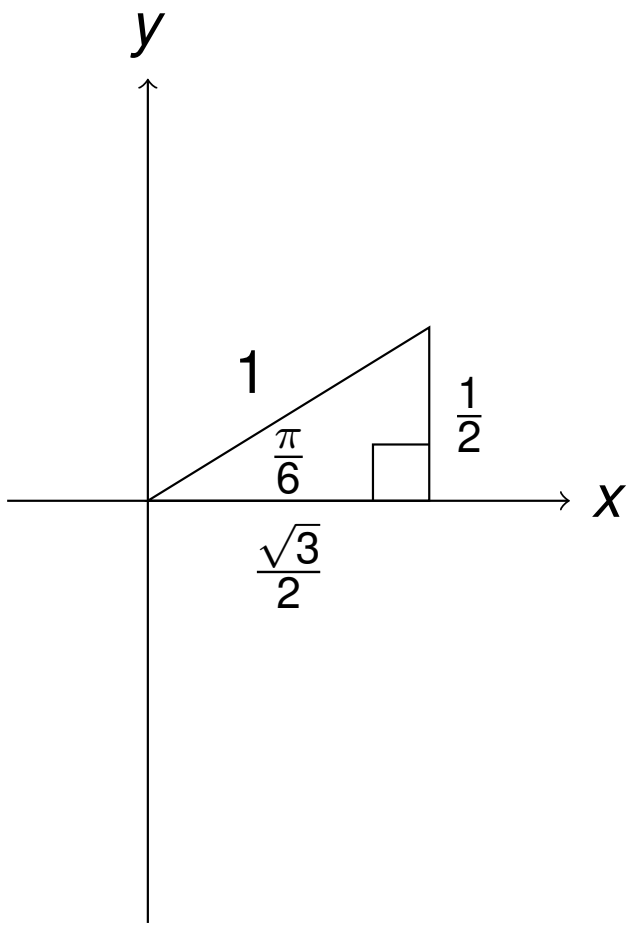
Example 5

Find

$$\cos^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{6}$$



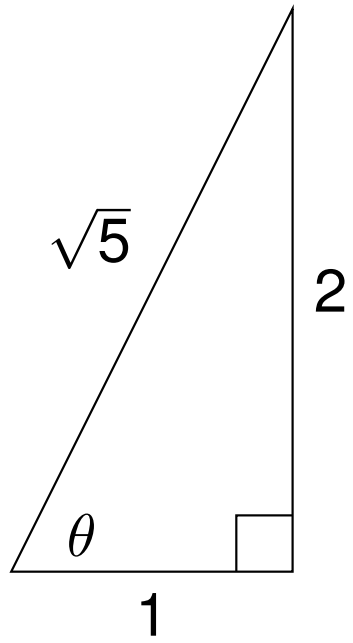
Example 5

Find $\cos\left(\tan^{-1} 2\right)$

$$\tan\theta = \frac{2}{1}$$

$$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\left(\tan^{-1} 2\right) = \frac{\sqrt{5}}{5}$$



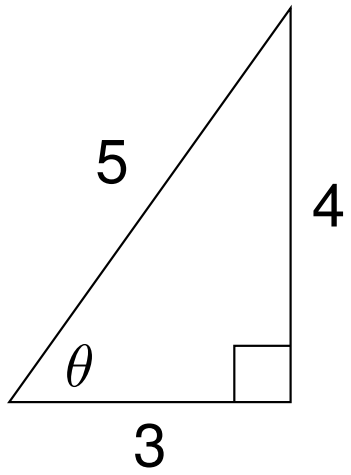
Example 6

Find $\tan\left(\cos^{-1}\frac{3}{5}\right)$

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

$$\tan\theta = \frac{4}{3}$$

$$\tan\left(\cos^{-1}\frac{3}{5}\right) = \frac{4}{3}$$



Example 7

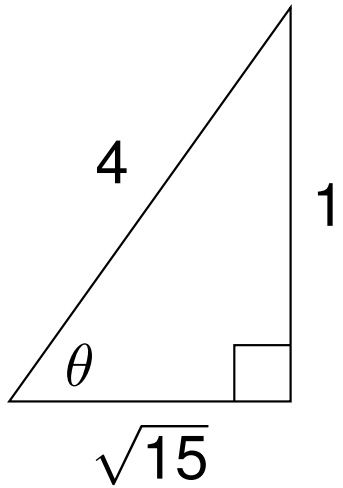
Find $\sec\left(\sin^{-1}\frac{1}{4}\right)$

$$\theta = \sin^{-1}\frac{1}{4}$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

$$\sec\theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

$$\sec\left(\sin^{-1}\frac{1}{4}\right) = \frac{4\sqrt{15}}{15}$$



Example 8

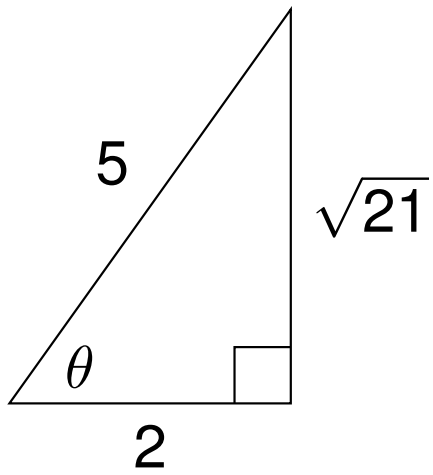
$$\tan\left(\sec^{-1}\frac{5}{2}\right)$$

$$= \tan\left(\cos^{-1}\frac{2}{5}\right)$$

$$\theta = \cos^{-1}\frac{2}{5}$$

$$\tan\theta = \frac{\sqrt{21}}{2}$$

$$\tan\left(\sec^{-1}\frac{5}{2}\right) = \frac{\sqrt{21}}{2}$$



Recap

- Inputs to inverse trig functions give information about the lengths of sides of a triangle.
- Label a triangle using this information.
- Find the missing side by the Pythagorean Theorem
- Read the values of the other trig functions from the triangle