Angle Sum Formulas

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries

- Be able to derive the six angle sum formulas
- Inverse trig functions
- Simplify fractions
- Rationalize the denominator

Objectives

- Use the angle sum formulas to find specific values

Angle Sum Formulas

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

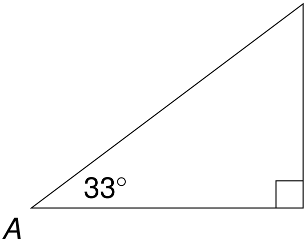
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

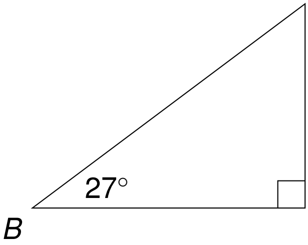
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Adding Angles

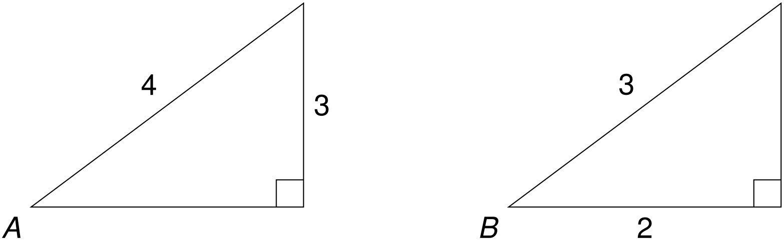
Find $\sin(A + B)$ if $A = 33^\circ$ and $B = 27^\circ$





Angles Not Given

Find $\sin(A + B)$ if $\sin A = \frac{3}{4}$ and $\cos B = \frac{2}{3}$



Angles Given but Value Unknown

Find $\sin(45^\circ + 30^\circ)$

$$\sin(45^\circ + 30^\circ) = \sin(75^\circ) = ???$$

Find $\sin(75^\circ)$

Solution:

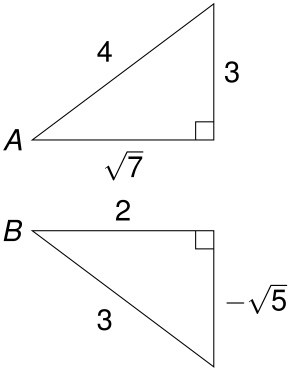
$$\begin{aligned}
\sin(75^\circ) &= \sin(45^\circ + 30^\circ) \\
&= \sin(45^\circ)\cos(30^\circ) + \cos(45^\circ)\sin(30^\circ) \\
&= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} \\
&= \frac{\sqrt{6} + \sqrt{2}}{4}
\end{aligned}$$

Example 1

Find $\sin(A+B)$ if $\sin A = \frac{3}{4}$ and $\cos B = \frac{2}{3}$, where A is in quadrant I and B is in quadrant IV.

$$\sin A = \frac{3}{4}, \; \cos A = \frac{\sqrt{7}}{4}$$

$$\cos B = \frac{2}{3}, \; \sin B = \frac{-\sqrt{5}}{3}$$



$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{\sqrt{7}}{4}\right)\left(-\frac{\sqrt{5}}{3}\right)$$

$$= \frac{6-\sqrt{35}}{12}$$

Example 2

Find $\sin(\sin^{-1}\frac{3}{4} - \cos^{-1}\frac{2}{3})$

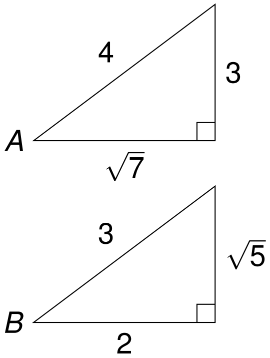
Let $A = \sin^{-1}\frac{3}{4}$ and let $B = \cos^{-1}\frac{2}{3}$ so that we are are asked to find $\sin(A - B)$

$$\sin A = \tfrac{3}{4},\ \cos A = \tfrac{\sqrt{7}}{4}$$
$$\cos B = \tfrac{2}{3},\ \sin B = \tfrac{\sqrt{5}}{3}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \left(\frac{3}{4}\right)\left(\frac{2}{3}\right) - \left(\frac{\sqrt{7}}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$$

$$= \frac{6 - \sqrt{35}}{12}$$

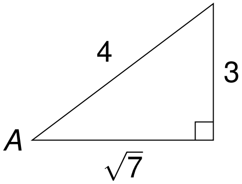


Example 3

Find $\tan(A - B)$ if $\sin A = \frac{3}{4}$ and $\cos B = \frac{2}{3}$, where A is in quadrant I and B is in quadrant IV.

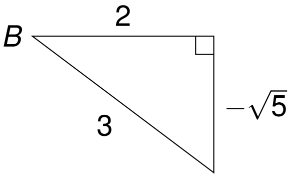
Solution:

$$\tan A = \frac{3}{\sqrt{7}}, \ \tan B = \frac{-\sqrt{5}}{2}$$



$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\left(\frac{3}{\sqrt{7}}\right) - \left(\frac{-\sqrt{5}}{2}\right)}{1 + \left(\frac{3}{\sqrt{7}}\right)\left(\frac{-\sqrt{5}}{2}\right)}$$



Example 3

$$\frac{\left(\frac{3}{\sqrt{7}}\right) - \left(\frac{-\sqrt{5}}{2}\right)}{1 + \left(\frac{3}{\sqrt{7}}\right)\left(\frac{-\sqrt{5}}{2}\right)} = \frac{\left(\frac{6}{2\sqrt{7}}\right) - \left(\frac{-\sqrt{35}}{2\sqrt{7}}\right)}{\left(\frac{2\sqrt{7}}{2\sqrt{7}}\right) - \left(\frac{3\sqrt{5}}{2\sqrt{7}}\right)}$$

$$= \frac{6 + \sqrt{35}}{2\sqrt{7} - 3\sqrt{5}} \left(\frac{2\sqrt{7} + 3\sqrt{5}}{2\sqrt{7} + 3\sqrt{5}}\right)$$

$$= \frac{12\sqrt{7} + 18\sqrt{5} + 14\sqrt{5} + 15\sqrt{7}}{28 - 45} = \frac{27\sqrt{7} + 32\sqrt{5}}{-17}$$

$$= -\frac{27\sqrt{7} + 32\sqrt{5}}{17} = \frac{-27\sqrt{7} - 32\sqrt{5}}{17}$$

Recap

- Draw and label triangles for each given trig value
- Use Pythagorean Theorem to find missing lengths
- Write down the appropriate formula
- Plug in values of trig functions from the triangles in steps 1 and 2
- Simplify