

## Values of Tangent and Inverse Tangent



## Preliminaries and Objectives

Preliminaries:

- Values of  $\sin x$  and  $\cos x$  from unit circle in both radians and degrees.
- Values of  $\sin^{-1} x$  and  $\cos^{-1} x$  from unit circle in both radians and degrees.

Objectives:

- Find unit circle values of the tangent and inverse tangent functions.

## Values of $\sin \theta$ , $\cos \theta$ and $\tan \theta$ - Quadrant I

$\theta$	$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	$0^\circ$	0	1	0
$\frac{\pi}{6}$	$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	$90^\circ$	1	0	undefined

## Values of $\sin \theta$ , $\cos \theta$ and $\tan \theta$ - Quadrants I and II

$\theta$	$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	$0^\circ$	0	1	0
$\frac{\pi}{6}$	$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	$90^\circ$	1	0	undefined
$\frac{2\pi}{3}$	$120^\circ$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	$135^\circ$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
$\frac{5\pi}{6}$	$150^\circ$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
$\pi$	$180^\circ$	0	-1	0

## Values of $\sin \theta$ , $\cos \theta$ and $\tan \theta$ - Quadrants III and IV

$\theta$	$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$\pi$	$180^\circ$	0	1	0
$\frac{7\pi}{6}$	$210^\circ$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{5\pi}{4}$	$225^\circ$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
$\frac{4\pi}{3}$	$240^\circ$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
$\frac{3\pi}{2}$	$270^\circ$	-1	0	undefined
$\frac{5\pi}{3}$	$300^\circ$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
$\frac{7\pi}{4}$	$315^\circ$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
$\frac{11\pi}{6}$	$330^\circ$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
$2\pi$	$360^\circ$	0	1	0

## Values of $\sin \theta$ , $\cos \theta$ and $\tan \theta$ - Negative Angles

$\theta$	$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$-\frac{\pi}{2}$	$-90^\circ$	-1	0	undefined
$-\frac{\pi}{3}$	$-60^\circ$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	$-45^\circ$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
$-\frac{\pi}{6}$	$-30^\circ$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
0	$0^\circ$	0	1	0
$\frac{\pi}{6}$	$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	$90^\circ$	1	0	undefined

## Values of $\tan^{-1} \theta$

$$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3} \quad \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\tan^{-1}(-1) = -\frac{\pi}{4} \quad \tan^{-1}(1) = \frac{\pi}{4}$$

$$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} \quad \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\tan^{-1}(0) = 0$$

## Recap

- To find  $\tan \theta$ , use  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .

- For  $y = \tan^{-1} x$ , the answer must be between  $-90^\circ$  and  $90^\circ$  (or in radians, between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ ).