

The Inverse Functions of Sine and Cosine



Preliminaries and Objectives

Preliminaries:

- Values of $\sin x$ and $\cos x$ from unit circle in both radians and degrees.

Objectives:

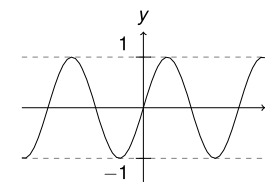
- Find values of the inverse sine and inverse cosine functions.

The sin Function

$$y = \sin x$$

x is an angle

$$-1 \leq y \leq 1$$



The sin Function

$$y = \sin x$$

x is an angle

$$-1 \leq y \leq 1$$

$$y = \sin^{-1} x$$

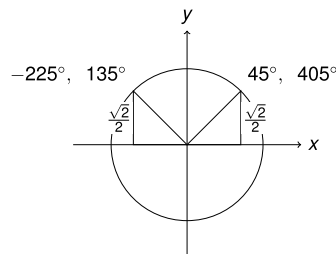
$$x = \sin y$$

y is the angle whose sin is x

$$-1 \leq x \leq 1$$

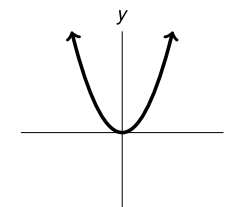
The Inverse Sine Function

$$y = \sin^{-1} \left(\frac{\sqrt{2}}{2} \right)$$



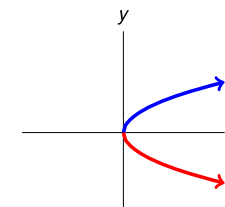
The Inverse of $y = x^2$

$$y = x^2$$



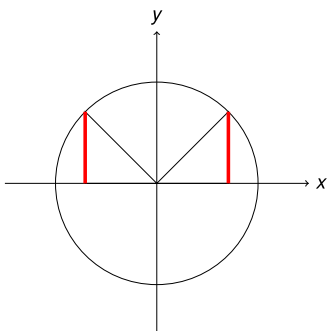
$$x = y^2$$

$$y = \pm\sqrt{x}$$



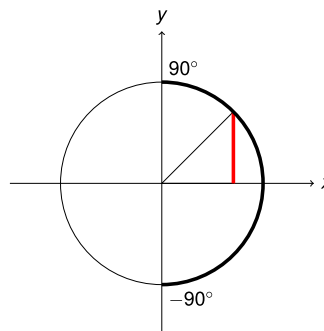
The Inverse Sine Function

$$\sin^{-1} \frac{\sqrt{2}}{2}$$



The Inverse Sine Function

$$\sin^{-1} \frac{\sqrt{2}}{2} = 45^\circ$$



The Inverse Sine Function

$$\sin^{-1}(-1) = -90^\circ$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ$$

$$\sin^{-1}(0) = 0^\circ$$

$$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

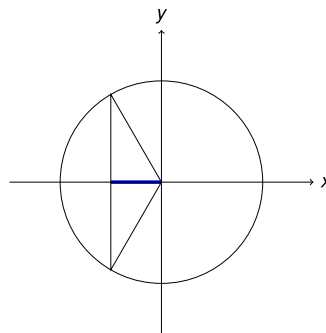
$$\sin^{-1}(1) = 90^\circ$$

The Inverse Sine Function

$$\begin{aligned} \sin^{-1}(-1) &= -\frac{\pi}{2} & \sin^{-1}(0) &= 0 \\ \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) &= -\frac{\pi}{3} & \sin^{-1}\left(\frac{1}{2}\right) &= \frac{\pi}{6} \\ \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) &= -\frac{\pi}{4} & \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) &= \frac{\pi}{4} \\ \sin^{-1}\left(-\frac{1}{2}\right) &= -\frac{\pi}{6} & \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) &= \frac{\pi}{3} \\ & & \sin^{-1}(1) &= \frac{\pi}{2} \end{aligned}$$

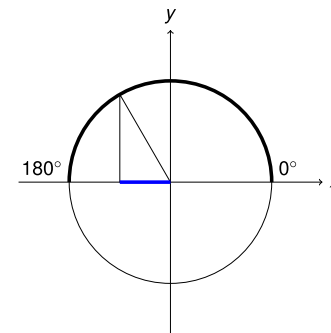
The Inverse Cosine Function

$$\cos^{-1} -\frac{1}{2}$$



The Inverse Cosine Function

$$\cos^{-1} -\frac{1}{2} = 120^\circ$$



The Inverse Cosine Function

$$\begin{aligned} \cos^{-1}(-1) &= 180^\circ & \cos^{-1}(0) &= 90^\circ \\ \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) &= 150^\circ & \cos^{-1}\left(\frac{1}{2}\right) &= 60^\circ \\ \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) &= 135^\circ & \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) &= 45^\circ \\ \cos^{-1}\left(-\frac{1}{2}\right) &= 120^\circ & \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) &= 30^\circ \\ & & \cos^{-1}(1) &= 0^\circ \end{aligned}$$

The Inverse Cosine Function

$$\begin{aligned} \cos^{-1}(-1) &= \pi & \cos^{-1}(0) &= \frac{\pi}{2} \\ \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) &= \frac{5\pi}{6} & \cos^{-1}\left(\frac{1}{2}\right) &= \frac{\pi}{3} \\ \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) &= \frac{3\pi}{4} & \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) &= \frac{\pi}{4} \\ \cos^{-1}\left(-\frac{1}{2}\right) &= \frac{2\pi}{3} & \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) &= \frac{\pi}{6} \\ & & \cos^{-1}(1) &= 0 \end{aligned}$$

Recap

- For $y = \sin^{-1} x$ and $y = \cos^{-1} x$, x is a number between -1 and 1 and y is an angle.
- For $y = \sin^{-1} x$, the answer must be between -90° and 90° .
- For $y = \cos^{-1} x$, the answer must be between 0° and 180° .