Solving Trig Equations - Part I

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Preliminaries and Objectives

Preliminaries
- Unit circle values in degrees and radians
- Inverse trig functions
- Algebraic techniques for solving polynomial equations

Objectives
- Find all solutions to a trigonometric equation.

Example 1

Find all angles θ such that

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \{ \ldots -120°, -60°, 240°, 300°, 600°, 660° \ldots \}$$

Example 1 in radians

Find all angles θ such that

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \left\{ \ldots -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \ldots \right\}$$

Example 2

Find all angles θ such that

$$3\sin \theta - 2 = -\frac{1}{2} \qquad\qquad 3x - 2 = -\frac{1}{2}$$

$$3\sin \theta = \frac{3}{2} \qquad\qquad 3x = \frac{3}{2}$$

$$\sin \theta = \frac{1}{2} \qquad\qquad x = \frac{1}{2}$$

$$\theta = \left\{ \ldots -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \ldots \right\}$$

Example 3

Find all angles θ such that

$$\tan^2 \theta = 3 \qquad\qquad x^2 = 3$$

$$\tan \theta = \pm\sqrt{3} \qquad\qquad x = \pm\sqrt{3}$$

$$\theta = \left\{ \ldots -\frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \ldots \right\}$$

Example 4

Find all angles θ such that

$$2\sin^2 \theta + \sin \theta - 1 = 0 \qquad\qquad 2x^2 + x - 1 = 0$$

$$(2\sin \theta - 1)(\sin \theta + 1) = 0 \qquad\qquad (2x - 1)(x + 1) = 0$$

$$(2\sin \theta - 1) = 0 \text{ or } (\sin \theta + 1) = 0 \qquad 2x - 1 = 0 \text{ or } x + 1 = 0$$

$$2\sin \theta = 1 \text{ or } \sin \theta = -1 \qquad\qquad 2x = 1 \text{ or } x = -1$$

$$\sin \theta = \tfrac{1}{2} \text{ or } \sin \theta = -1 \qquad\qquad x = \tfrac{1}{2} \text{ or } x = -1$$

$$\theta = \left\{ \ldots -\frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{7\pi}{2} \ldots \right\}$$

Recap

- Solve the trig equation to find the numerical values for the trig functions
- Look up the angles from the unit circle
- Add and subtract full circles as necessary