**Parametric Equations**

- **Preliminaries and Objectives**
  - Preliminaries
    - Equations of lines, including point-slope form
    - Equations of circles and ellipses
    - The sin and cos functions and the unit circle
  - Objectives
    - Analyze functions and graphs where $x$ and $y$ are defined as functions of time.

- **Graph of a Circle**
  - $y = \sqrt{1-x^2}$
  - $y = -\sqrt{1-x^2}$

- **Graph of a Circle**
  - $x(t) = \cos t$
  - $y(t) = \sin t$

- **Parametric Equation of an Ellipse**
  - $x = 5 \cos t$
  - $y = 3 \sin t$

- **Parametric Equation of a Circle**
  - $x = \cos t; y = \sin t$
    - for $0 \leq t \leq 2\pi$
  - $x = \cos 2t; y = \sin 2t$
    - for $0 \leq t \leq \pi$
  - $x = \sin t; y = \cos t$
    - for $0 \leq t \leq 2\pi$
  - $x = \cos t; y = \sin t$
    - for $0 \leq t \leq 4\pi$

- **Parametric Equation of an Ellipse**
  - $x = 2 + 4 \cos t$
  - $y = 3 + 2 \sin t$

- **Parametric Equation of a Line**
  - $x = 1 + 3t$
  - $y = -2 + 5t$
  - $m = \frac{5}{3}$
  - Goes through the point $(1, -2)$
  - Point-slope form: $(y + 2) = \frac{5}{3}(x - 1)$
Parametric Equation of a Line Segment

Write the parametric equations of a line segment that begins at the point $(3, -2)$ at time $t = 0$ and ends at the point $(-12, 8)$ at time $t = 5$.

Solution: The point moves a distance of $-12 - 3 = -15$ in the $x$-direction, so the speed in the $x$-direction is $\frac{15}{5} = -3$. The point moves a distance of $8 - (-2) = 10$ in the $y$-direction, so the speed in the $y$-direction is $\frac{10}{5} = 2$. The equations are therefore

\[
x(t) = -3t + 3
\]
\[
y(t) = 2t - 2
\]
for $0 \leq t \leq 5$

Connected $(1, -2)$ to $(13, 18)$

Parametric Form of Functions

\[
x = t; \quad y = t^2
\]
\[
y = x^2
\]

Functions from Parametric Form

Solve one variable for $t$, substitute in other variable equation.

\[
x = 3t^2 + 4; \quad y = 2t - 4
\]
\[
\frac{y + 4}{2} = t
\]
\[
x = 3\left(\frac{y + 4}{2}\right)^2 + 4
\]
\[
x - 4 = \frac{3}{4}(y + 4)^2
\]

Recap

- Plot a parametric graph by picking values for $t$ to find points $(x, y)$.
- Given $y = f(x)$, then let $x = t$ to get parametric equations.
- Given parametric equations, solve for $t$ and substitute into other equation to get $y = f(x)$. 

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