Parametric Equations

University of Minnesota

Preliminaries and Objectives

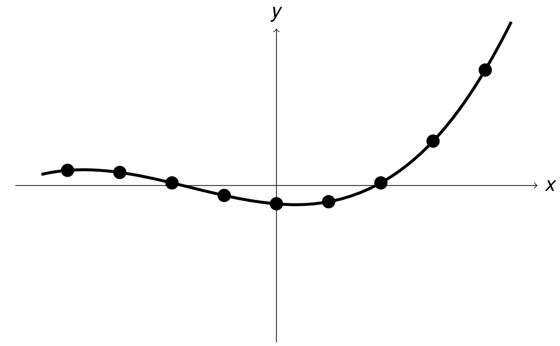
Preliminaries

- Equations of lines, including point-slope form
- Equations of circles and ellipses
- The *sin* and *cos* functions and the unit circle

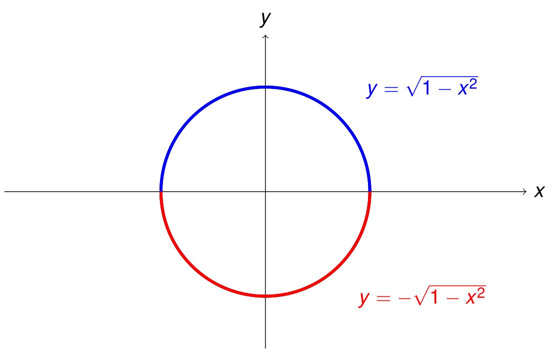
Objectives

- Analyze functions and graphs where x and y are defined as functions of time.

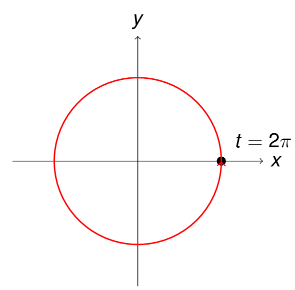
Graphs of Functions



Graph of a Circle



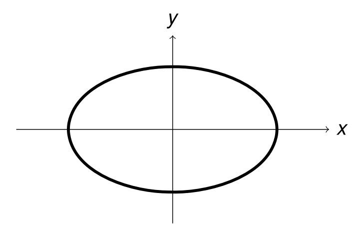
Graph of a Circle



$x(t) = \cos t \qquad y(t) = \sin t$

Parametric Equation of an Ellipse

$x = 5\cos t \qquad y = 3\sin t$



$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Parametric Equation of a Circle

$x = \cos t; \qquad y = \sin t$
for $0 \le t \le 2\pi$

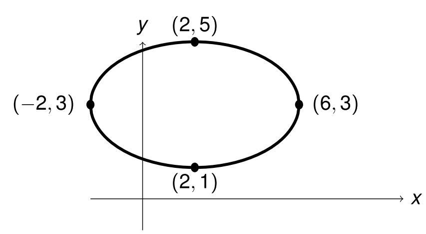
$x = \cos 2t; \qquad y = \sin 2t$
for $0 \le t \le \pi$

$x = \sin t; \qquad y = \cos t$
for $0 \le t \le 2\pi$

$x = \cos t; \qquad y = \sin t$
for $0 \le t \le 4\pi$

Parametric Equation of an Ellipse

$x = 2 + 4\cos t \qquad y = 3 + 2\sin t$



Parametric Equation of a Line

$x = 1 + 3t; \qquad y = -2 + 5t$

$m = \frac{5}{3}$ Goes through the point $(1, -2)$

Point-slope form: $(y + 2) = \frac{5}{3}(x - 1)$

Parametric Equation of a Line Segment

$$x = 1 + 3t; \qquad y = -2 + 5t$$

$$0 \le t \le 4$$

Connects $(1, -2)$ to $(13, 18)$

Parametric Equation of a Line Segment

Write the parametric equations of a line segment that begins at the point $(3, -2)$ at time $t = 0$ and ends at the point $(-12, 8)$ at time $t = 5$.

Solution: The point moves a distance of $-12 - 3 = -15$ in the x-direction, so the speed in the x-direction is $\frac{-15}{5} = -3$. The point moves a distance of $8 - (-2) = 10$ in the y-direction, so the speed in the y-direction is $\frac{10}{5} = 2$. The equations are therefore

$$x(t) = -3t + 3$$

$$y(t) = 2t - 2$$

for $0 \le t \le 5$

Parametric Form of Functions

$$x = t; \qquad y = t^2$$

$$y = x^2$$

Functions from Parametric Form

Solve one variable for t, substitute in other variable equation.

$$x = 3t^2 + 4 \qquad y = 2t - 4$$

$$\frac{y+4}{2} = t$$

$$x = 3\left(\frac{y+4}{2}\right)^2 + 4$$

$$x - 4 = \frac{3}{4}(y+4)^2$$

Recap

- Plot a parametric graph by picking values for t to find points (x, y).
- Given $y = f(x)$, then let $x = t$ to get parametric equations.
- Given parametric equations, solve for t and substitute into other equation to get $y = f(x)$.