

General Equation of an Ellipse



Preliminaries and Objectives

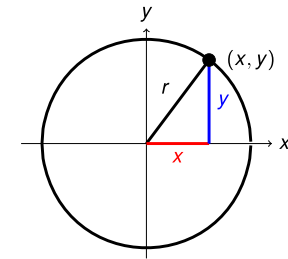
Preliminaries

- Equation of a circle
- Transformation of graphs (shifting and stretching)

Objectives

- Find the equation of an ellipse, given the graph.

Circle centered at the origin



$$x^2 + y^2 = r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \quad \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Stretching, Period and Wavelength

$$y = \sin(Bx)$$

The sine wave is B times thinner. Period (wavelength) is divided by B . Frequency is multiplied by B .

$$y = \sin\left(\frac{x}{b}\right)$$

The sine wave is b times wider. Period (wavelength) is multiplied by b . Frequency is divided by b .

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

The unit circle is stretched r times wider and r times taller.

Ellipse Centered at the Origin

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

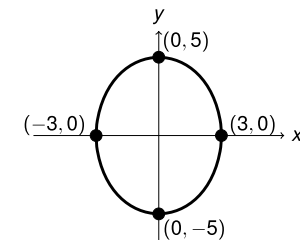
The unit circle is stretched r times wider and r times taller.

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

The unit circle is stretched a times wider and b times taller.

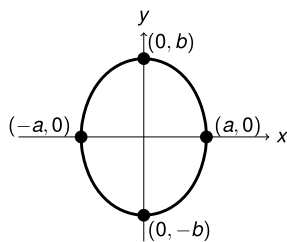
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse centered at the origin



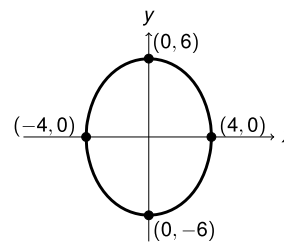
$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

Ellipse centered at the origin



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse centered at the origin



$$\frac{x^2}{16} + \frac{y^2}{36} = 1$$

General Form of an Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

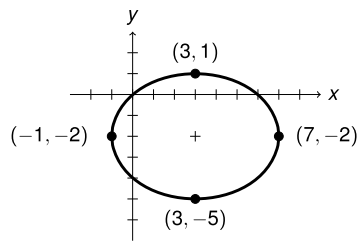
Center at (h, k)

Vertices at $(h+a, k)$, $(h-a, k)$, $(h, k+b)$, $(h, k-b)$

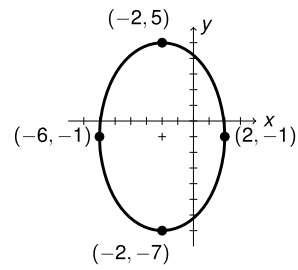
Example 1

Graph $9(x - 3)^2 + 16(y + 2)^2 = 144$

$$\frac{(x - 3)^2}{16} + \frac{(y + 2)^2}{9} = 1$$



Example 2



$$\frac{(x + 2)^2}{16} + \frac{(y + 1)^2}{36} = 1$$

Recap

General Equation of an Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Center at (h, k)

Vertices at $(h + a, k)$, $(h - a, k)$, $(h, k + b)$, $(h, k - b)$