

General Equation of a Hyperbola



Preliminaries and Objectives

Preliminaries

- Transformation of graphs (shifting and stretching)

Objectives

- Graph a hyperbola, given the equation.
- Find the equation of a hyperbola, given the graph.

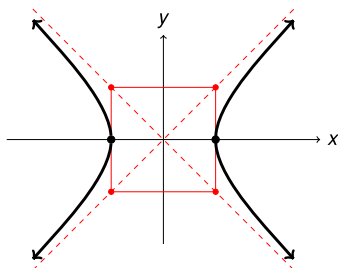
Ellipse Centered at the Origin

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The unit circle is stretched a times wider and b times taller.

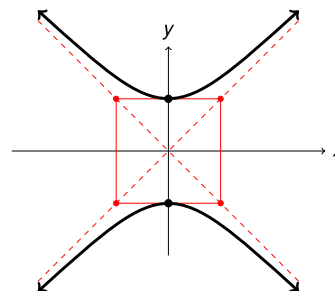
Standard Hyperbola

$$x^2 - y^2 = 1$$



Standard Hyperbola - Vertical

$$y^2 - x^2 = 1$$



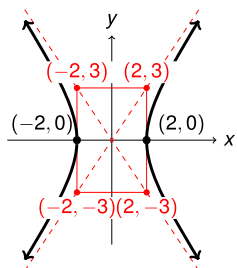
Stretched Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The hyperbola is stretched a times wider and b times taller.

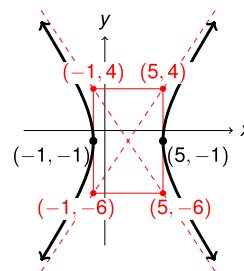
Stretched Hyperbola

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$



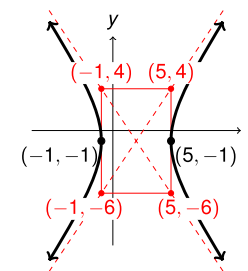
Writing the equation from the graph

$$\frac{(\quad)^2}{\quad} - \frac{(\quad)^2}{\quad} = 1$$



Writing the equation from the graph

$$\frac{(x-2)^2}{9} - \frac{(y+1)^2}{25} = 1$$



Center at $(2, -1)$

Recap

General Equation of a Hyperbola- Horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center at (h, k)

Asymptotes have slope $\pm \frac{b}{a}$ and pass through the center

Vertices at $(h + a, k), (h - a, k)$

Recap

General Equation of a Hyperbola - Vertical

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Center at (h, k)

Asymptotes have slope $\pm \frac{b}{a}$ and pass through the center

Vertices at $(h, k + b), (h, k - b)$