

Double and Half Angle Formulas



Preliminaries and Objectives

Preliminaries

- Be able to derive the double angle formulas from the angle sum formulas
- Inverse trig functions
- Simplify fractions
- Rationalize the denominator

Objectives

- Use the double angle formulas to find specific values

Double and Half Angle Formulas

$$\begin{aligned} \sin(2A) &= 2 \sin A \cos A & \sin\left(\frac{A}{2}\right) &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos(2A) &= \cos^2 A - \sin^2 A & \cos\left(\frac{A}{2}\right) &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ &= 2 \cos^2 A - 1 & \tan\left(\frac{A}{2}\right) &= \frac{\sin A}{1 + \cos A} \\ &= 1 - 2 \sin^2 A & &= \frac{1 - \cos A}{\sin A} \\ \tan(2A) &= \frac{2 \tan A}{1 - \tan^2 A} & & \end{aligned}$$

Double and Half Angle Formulas

Find $\sin(2\theta)$, $\tan(2\theta)$ and $\tan(\frac{\theta}{2})$ if $\tan \theta = \frac{3}{2}$, where $\pi < \theta < \frac{3\pi}{2}$

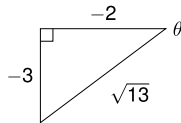
Solution:

$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{-3}{\sqrt{13}} \right) \left(\frac{-2}{\sqrt{13}} \right)$$

$$= \frac{12}{13}$$



Double and Half Angle Formulas

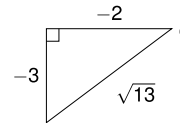
Find $\sin(2\theta)$, $\tan(2\theta)$ and $\tan(\frac{\theta}{2})$ if $\tan \theta = \frac{3}{2}$, where $\pi < \theta < \frac{3\pi}{2}$

Solution:

$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2(\frac{3}{2})}{1 - (\frac{3}{2})^2} = \frac{3}{-\frac{5}{4}} = -\frac{12}{5}$$



Double and Half Angle Formulas

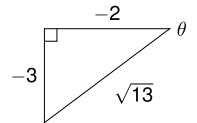
Find $\sin(2\theta)$, $\tan(2\theta)$ and $\tan(\frac{\theta}{2})$ if $\tan \theta = \frac{3}{2}$, where $\pi < \theta < \frac{3\pi}{2}$

Solution:

$$\sin \theta = \frac{-3}{\sqrt{13}}, \cos \theta = \frac{-2}{\sqrt{13}}, \tan \theta = \frac{3}{2}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin \theta}{1 + \cos \theta}$$

$$= \frac{-\frac{3}{\sqrt{13}}}{1 - \frac{2}{\sqrt{13}}} = \frac{-2 - \sqrt{13}}{3}$$



Previous Answer with slower algebra

$$\frac{-\frac{3}{\sqrt{13}}}{1 - \frac{2}{\sqrt{13}}} = \frac{-\frac{3}{\sqrt{13}} \cdot \sqrt{13}}{\left(1 - \frac{2}{\sqrt{13}}\right) \cdot \sqrt{13}} = \frac{-3}{\sqrt{13} - 2}$$

$$= \frac{-3(\sqrt{13} + 2)}{(\sqrt{13} - 2)(\sqrt{13} + 2)} = \frac{-3\sqrt{13} - 6}{13 - 4}$$

$$= \frac{3(-\sqrt{13} - 2)}{9} = \frac{-2 - \sqrt{13}}{3}$$

Double and Half Angle Formulas

Find $\sin(2 \tan^{-1} \frac{3}{2})$

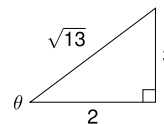
Solution:

Let $\theta = \tan^{-1} \frac{3}{2}$ so that we are asked to find $\sin(2\theta)$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\sin \theta = \frac{3}{\sqrt{13}}, \cos \theta = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \sin(2\theta) = 2 \left(\frac{3}{\sqrt{13}} \right) \left(\frac{2}{\sqrt{13}} \right) = \frac{12}{13}$$



Recap

- Draw and label triangles for each given trig value
- Use Pythagorean Theorem to find missing lengths
- Write down the appropriate formula
- Plug in values of trig functions from the triangles in steps 1 and 2
- Simplify