Combining Trig Functions and Inverse Trig Functions - Part II

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Preliminaries and Objectives

Preliminaries:

- Trig functions
- Inverse Trig Functions
- Angle Sum Formulas

Objectives:

- Find values when trig functions and inverse trig functions are combined in angle sum formulas.

Angle Sum Formulas - Example 1

Find

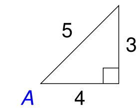
$\sin\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{2}{3}\right) \Leftrightarrow \sin\left(A+B\right)$

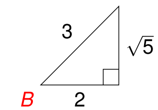
$\tan A=\frac{3}{4}$; $\sin A=\frac{3}{5}$; $\cos A=\frac{4}{5}$

$\cos B=\frac{2}{3}$; $\sin B=\frac{\sqrt{5}}{3}$; $\tan B=\frac{\sqrt{5}}{2}$

$\sin\left(A+B\right)=(\sin A)(\cos B)+(\cos A)(\sin B)$

$$=\left(\frac{3}{5}\right)\left(\frac{2}{3}\right)+\left(\frac{4}{5}\right)\left(\frac{\sqrt{5}}{3}\right)=\frac{6+4\sqrt{5}}{15}$$



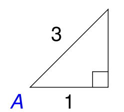


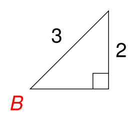
Angle Sum Formulas - Example 2

Find

$\cos\left(\sec^{-1}3+\sin^{-1}\frac{2}{3}\right) \Leftrightarrow \cos\left(A+B\right)$

$\sec A=3$; $\cos A=\frac{1}{3}$





Angle Sum Formulas - Example 3

To find

$\csc\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{2}{3}\right) \Leftrightarrow \csc\left(A+B\right) \Leftrightarrow \frac{1}{\sin\left(A+B\right)}$

first find

$\sin\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{2}{3}\right)=\frac{6+4\sqrt{5}}{15}$

then take the reciprocal to get

$\csc\left(\tan^{-1}\frac{3}{4}+\cos^{-1}\frac{2}{3}\right)=\frac{15}{6+4\sqrt{5}}=\frac{30\sqrt{5}-45}{22}$

Recap

- Inverse trig functions give information about two sides of a triangle
- The third side can be found by the Pythagorean Theorem
- Values of trig functions can be found from the triangle and plugged into the appropriate formula.

This technique will also work for double and half angle formulas.

Angle Sum Formulas - Solution to Example 2

In case you wanted the answer to the second example:

$\cos\left(\sec^{-1}3+\sin^{-1}\frac{2}{3}\right) \Leftrightarrow \cos\left(A+B\right)$

$\sec A=3$; $\cos A=\frac{1}{3}$; $\sin A=\frac{2\sqrt{2}}{3}$

$\sin B=\frac{2}{3}$; $\cos B=\frac{\sqrt{5}}{3}$

$\cos\left(\sec^{-1}3+\sin^{-1}\frac{2}{3}\right)=(\frac{1}{3})(\frac{\sqrt{5}}{3})-(\frac{2\sqrt{2}}{3})(\frac{2}{3})=\frac{\sqrt{5}-4\sqrt{2}}{9}$