

## Angle Sum Formulas



## Preliminaries and Objectives

### Preliminaries

- Be able to derive the six angle sum formulas
- Inverse trig functions
- Simplify fractions
- Rationalize the denominator

### Objectives

- Use the angle sum formulas to find specific values

## Angle Sum Formulas

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

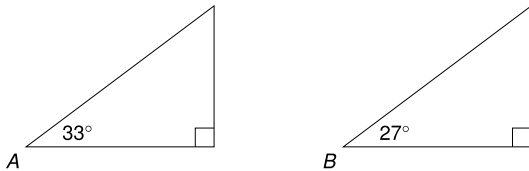
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

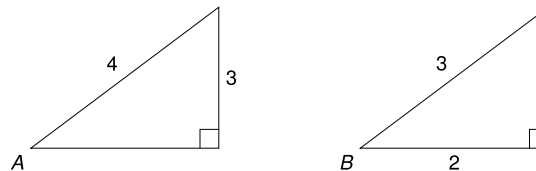
## Adding Angles

Find  $\sin(A + B)$  if  $A = 33^\circ$  and  $B = 27^\circ$



## Angles Not Given

Find  $\sin(A + B)$  if  $\sin A = \frac{3}{4}$  and  $\cos B = \frac{2}{3}$



## Angles Given but Value Unknown

Find  $\sin(45^\circ + 30^\circ)$

$$\sin(45^\circ + 30^\circ) = \sin(75^\circ) = ???$$

## $\sin(A + B) = \sin A \cos B + \cos A \sin B$

Find  $\sin(75^\circ)$

Solution:

$$\begin{aligned} \sin(75^\circ) &= \sin(45^\circ + 30^\circ) \\ &= \sin(45^\circ) \cos(30^\circ) + \cos(45^\circ) \sin(30^\circ) \\ &= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

## Example 1

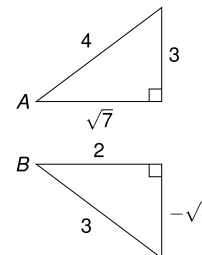
Find  $\sin(A + B)$  if  $\sin A = \frac{3}{4}$  and  $\cos B = \frac{2}{3}$ , where  $A$  is in quadrant I and  $B$  is in quadrant IV.

$$\sin A = \frac{3}{4}, \cos A = \frac{\sqrt{7}}{4}$$

$$\cos B = \frac{2}{3}, \sin B = \frac{-\sqrt{5}}{3}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\begin{aligned} &= \left(\frac{3}{4}\right) \left(\frac{2}{3}\right) + \left(\frac{\sqrt{7}}{4}\right) \left(\frac{-\sqrt{5}}{3}\right) \\ &= \frac{6 - \sqrt{35}}{12} \end{aligned}$$



## Example 2

Find  $\sin(\sin^{-1} \frac{3}{4} - \cos^{-1} \frac{2}{3})$

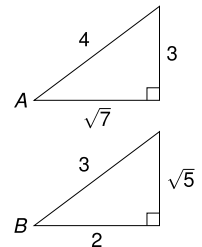
Let  $A = \sin^{-1} \frac{3}{4}$  and let  $B = \cos^{-1} \frac{2}{3}$  so that we are asked to find  $\sin(A - B)$

$$\sin A = \frac{3}{4}, \cos A = \frac{\sqrt{7}}{4}$$

$$\cos B = \frac{2}{3}, \sin B = \frac{\sqrt{5}}{3}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\begin{aligned} &= \left(\frac{3}{4}\right) \left(\frac{2}{3}\right) - \left(\frac{\sqrt{7}}{4}\right) \left(\frac{\sqrt{5}}{3}\right) \\ &= \frac{6 - \sqrt{35}}{12} \end{aligned}$$



### Example 3

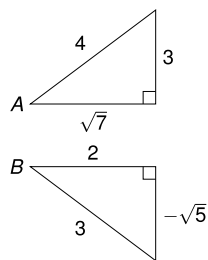
Find  $\tan(A - B)$  if  $\sin A = \frac{3}{4}$  and  $\cos B = \frac{2}{3}$ , where  $A$  is in quadrant I and  $B$  is in quadrant IV.

Solution:

$$\tan A = \frac{3}{\sqrt{7}}, \quad \tan B = \frac{-\sqrt{5}}{2}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\left(\frac{3}{\sqrt{7}}\right) - \left(\frac{-\sqrt{5}}{2}\right)}{1 + \left(\frac{3}{\sqrt{7}}\right)\left(\frac{-\sqrt{5}}{2}\right)}$$



### Example 3

$$\frac{\left(\frac{3}{\sqrt{7}}\right) - \left(\frac{-\sqrt{5}}{2}\right)}{1 + \left(\frac{3}{\sqrt{7}}\right)\left(\frac{-\sqrt{5}}{2}\right)} = \frac{\left(\frac{6}{2\sqrt{7}}\right) - \left(\frac{-\sqrt{35}}{2\sqrt{7}}\right)}{\left(\frac{2\sqrt{7}}{2\sqrt{7}}\right) - \left(\frac{3\sqrt{5}}{2\sqrt{7}}\right)}$$

$$= \frac{6 + \sqrt{35}}{2\sqrt{7} - 3\sqrt{5}} \left(\frac{2\sqrt{7} + 3\sqrt{5}}{2\sqrt{7} + 3\sqrt{5}}\right)$$

$$= \frac{12\sqrt{7} + 18\sqrt{5} + 14\sqrt{5} + 15\sqrt{7}}{28 - 45} = \frac{27\sqrt{7} + 32\sqrt{5}}{-17}$$

$$= -\frac{27\sqrt{7} + 32\sqrt{5}}{17} = \frac{-27\sqrt{7} - 32\sqrt{5}}{17}$$

### Recap

- Draw and label triangles for each given trig value
- Use Pythagorean Theorem to find missing lengths
- Write down the appropriate formula
- Plug in values of trig functions from the triangles in steps 1 and 2
- Simplify