

Values of Tangent and Inverse Tangent



Preliminaries and Objectives

Preliminaries:

- Values of $\sin x$ and $\cos x$ from unit circle in both radians and degrees.
- Values of $\sin^{-1} x$ and $\cos^{-1} x$ from unit circle in both radians and degrees.

Objectives:

- Find unit circle values of the tangent and inverse tangent functions.

Values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ - Quadrant I

θ	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0°	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	undefined

Values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ - Quadrants I and II

θ	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0°	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	undefined
$\frac{2\pi}{3}$	120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
$\frac{5\pi}{6}$	150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
π	180°	0	-1	0

Values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ - Quadrants III and IV

θ	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
π	180°	0	1	0
$\frac{7\pi}{6}$	210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{5\pi}{4}$	225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
$\frac{4\pi}{3}$	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
$\frac{3\pi}{2}$	270°	-1	0	undefined
$\frac{5\pi}{3}$	300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
$\frac{7\pi}{4}$	315°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
$\frac{11\pi}{6}$	330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
2π	360°	0	1	0

Values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ - Negative Angles

θ	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
$-\frac{\pi}{2}$	-90°	-1	0	undefined
$-\frac{\pi}{3}$	-60°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-45°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
$-\frac{\pi}{6}$	-30°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
0	0°	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	undefined

Values of $\tan^{-1} \theta$

$$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3} \quad \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\tan^{-1}(-1) = -\frac{\pi}{4} \quad \tan^{-1}(1) = \frac{\pi}{4}$$

$$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} \quad \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\tan^{-1}(0) = 0$$

Recap

- To find $\tan \theta$, use $\tan \theta = \frac{\sin \theta}{\cos \theta}$.
- For $y = \tan^{-1} x$, the answer must be between -90° and 90° (or in radians, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$).