

Solving Trig Equations - Part II



Preliminaries and Objectives

Preliminaries

- Unit circle values in degrees and radians
- Inverse trig functions
- Algebraic techniques for solving polynomial equations

Objectives

- Find all solutions to a trigonometric equation.

Example 1

Find all angles θ such that

$$\sin(3\theta) = -\frac{\sqrt{3}}{2}$$

$$3\theta = \{\dots - 120^\circ, -60^\circ, 240^\circ, 300^\circ, 600^\circ, 660^\circ \dots\}$$

$$\frac{3\theta}{3} = \{\dots, \frac{-120^\circ}{3}, \frac{-60^\circ}{3}, \frac{240^\circ}{3}, \frac{300^\circ}{3}, \frac{600^\circ}{3}, \frac{660^\circ}{3}, \dots\}$$

$$\theta = \{\dots - 40^\circ, -20^\circ, 80^\circ, 100^\circ, 200^\circ, 220^\circ \dots\}$$

Example 2

Find all angles θ such that

$$\sec 2\theta = 2$$

$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = \{\dots - 300^\circ, -60^\circ, 60^\circ, 300^\circ, 420^\circ, 660^\circ \dots\}$$

$$\frac{2\theta}{2} = \{\dots \frac{-300^\circ}{2}, \frac{-60^\circ}{2}, \frac{60^\circ}{2}, \frac{300^\circ}{2}, \frac{420^\circ}{2}, \frac{660^\circ}{2}, \dots\}$$

$$\theta = \{\dots - 150^\circ, -30^\circ, 30^\circ, 150^\circ, 210^\circ, 330^\circ \dots\}$$

Example 3

Find all angles θ such that

$$\tan\left(\theta - \frac{\pi}{2}\right) = -1$$

$$\theta - \frac{\pi}{2} = \left\{ \dots - \frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4} \dots \right\}$$

$$\theta = \left\{ \dots - \frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4} \dots \right\}$$

Example 4

Find all angles θ such that

$$\sin \theta = 2$$

There is no solution, since $-1 \leq \sin \theta \leq 1$

Recap

- Solve the trig equation to find the numerical values for the trig functions
- Look up the angles from the unit circle
- Add or subtract full circles as necessary
- Solve for θ