Solving Trig Equations - Part I

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Preliminaries and Objectives

Preliminaries

- Unit circle values in degrees and radians
- Inverse trig functions
- Algebraic techniques for solving polynomial equations

Objectives

- Find all solutions to a trigonometric equation.

Example 1

Find all angles θ such that

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \{\ldots -120°, -60°, 240°, 300°, 600°, 660° \ldots\}$$

Example 1 in radians

Find all angles θ such that

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \left\{ \ldots -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \ldots \right\}$$

Example 2

Find all angles θ such that

$$3\sin\theta - 2 = -\tfrac{1}{2} \qquad\qquad 3x - 2 = -\tfrac{1}{2}$$

$$3\sin\theta = \tfrac{3}{2} \qquad\qquad 3x = \tfrac{3}{2}$$

$$\sin\theta = \tfrac{1}{2} \qquad\qquad x = \tfrac{1}{2}$$

$$\theta = \left\{ \ldots -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \cdots \right\}$$

Example 3

Find all angles θ such that

$$\tan^2\theta = 3 \qquad\qquad x^2 = 3$$

$$\tan\theta = \pm\sqrt{3} \qquad\qquad x = \pm\sqrt{3}$$

$$\theta = \left\{ \ldots -\frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \cdots \right\}$$

Example 4

Find all angles θ such that

$$2\sin^2\theta + \sin\theta - 1 = 0 \qquad\qquad 2x^2 + x - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0 \qquad\qquad (2x - 1)(x + 1) = 0$$

$$(2\sin\theta - 1) = 0 \text{ or } (\sin\theta + 1) = 0 \qquad 2x - 1 = 0 \text{ or } x + 1 = 0$$

$$2\sin\theta = 1 \text{ or } \sin\theta = -1 \qquad\qquad 2x = 1 \text{ or } x = -1$$

$$\sin\theta = \tfrac{1}{2} \text{ or } \sin\theta = -1 \qquad\qquad x = \tfrac{1}{2} \text{ or } x = -1$$

$$\theta = \left\{ \ldots -\frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{7\pi}{2} \cdots \right\}$$

Recap

- Solve the trig equation to find the numerical values for the trig functions
- Look up the angles from the unit circle
- Add and subtract full circles as necessary