### Preliminaries and Objectives

**Preliminaries**
- Equations of lines, including point-slope form
- Equations of circles and ellipses
- The $\sin$ and $\cos$ functions and the unit circle

**Objectives**
- Analyze functions and graphs where $x$ and $y$ are defined as functions of time.

---

### Graphs of Functions

![Graph of a Function](image)

### Graph of a Circle

![Graph of a Circle](image)

$$y = \sqrt{1 - x^2}$$

$$y = -\sqrt{1 - x^2}$$
Graph of a Circle

\[ x(t) = \cos t \quad y(t) = \sin t \]

Parametric Equation of a Circle

\[
\begin{align*}
x &= \cos t; \quad &y &= \sin t \\
&\text{for } 0 \leq t \leq 2\pi
\end{align*}
\]

\[
\begin{align*}
x &= \cos 2t; \quad &y &= \sin 2t \\
&\text{for } 0 \leq t \leq \pi
\end{align*}
\]

\[
\begin{align*}
x &= \sin t; \quad &y &= \cos t \\
&\text{for } 0 \leq t \leq 2\pi
\end{align*}
\]

\[
\begin{align*}
x &= \cos t; \quad &y &= \sin t \\
&\text{for } 0 \leq t \leq 4\pi
\end{align*}
\]

Parametric Equation of an Ellipse

\[
x = 5 \cos t \quad y = 3 \sin t
\]

\[
\frac{x^2}{25} + \frac{y^2}{9} = 1
\]

Parametric Equation of an Ellipse

\[
x = 2 + 4 \cos t \quad y = 3 + 2 \sin t
\]

\[
\begin{align*}
(2, 5) \\
(-2, 3) \\
(2, 1) \\
(6, 3)
\end{align*}
\]
**Parametric Equation of a Line**

\[ x = 1 + 3t; \quad y = -2 + 5t \]

\[ m = \frac{5}{3} \quad \text{Goes through the point \((1, -2)\)} \]

Point-slope form: \( (y + 2) = \frac{5}{3}(x - 1) \)

---

**Parametric Equation of a Line Segment**

\[ x = 1 + 3t; \quad y = -2 + 5t \]

\[ 0 \leq t \leq 4 \]

Connects \((1, -2)\) to \((13, 18)\)

---

**Parametric Equation of a Line Segment**

Write the parametric equations of a line segment that begins at the point \((3, -2)\) at time \(t = 0\) and ends at the point \((-12, 8)\) at time \(t = 5\).

Solution: The point moves a distance of \(-12 - 3 = -15\) in the \(x\)-direction, so the speed in the \(x\)-direction is \(-\frac{15}{5} = -3\). The point moves a distance of \(8 - (-2) = 10\) in the \(y\)-direction, so the speed in the \(y\)-direction is \(\frac{10}{5} = 2\). The equations are therefore

\[ x(t) = -3t + 3 \]
\[ y(t) = 2t - 2 \]
for \(0 \leq t \leq 5\)

---

**Parametric Form of Functions**

\[ x = t; \quad y = t^2 \]

\[ y = x^2 \]
Functions from Parametric Form

Solve one variable for \( t \), substitute in other variable equation.

\[
x = 3t^2 + 4 \quad y = 2t - 4
\]

\[
\frac{y + 4}{2} = t
\]

\[
x = 3 \left( \frac{y + 4}{2} \right)^2 + 4
\]

\[
x - 4 = \frac{3}{4}(y + 4)^2
\]

Recap

- Plot a parametric graph by picking values for \( t \) to find points \((x, y)\).
- Given \( y = f(x) \), then let \( x = t \) to get parametric equations.
- Given parametric equations, solve for \( t \) and substitute into other equation to get \( y = f(x) \).