Preliminaries and Objectives

Preliminaries
- Equation of a circle
- Transformation of graphs (shifting and stretching)

Objectives
- Find the equation of an ellipse, given the graph.

Circle centered at the origin

\[
x^2 + y^2 = r^2
\]

\[
\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1
\]

\[
\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1
\]

Stretching, Period and Wavelength

\[
y = \sin(Bx)
\]

The sine wave is \(B\) times thinner. Period (wavelength) is divided by \(B\). Frequency is multiplied by \(B\).

\[
y = \sin\left(\frac{x}{b}\right)
\]

The sine wave is \(b\) times wider. Period (wavelength) is multiplied by \(b\). Frequency is divided by \(b\).

\[
\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1
\]

The unit circle is stretched \(r\) times wider and \(r\) times taller.
Ellipse Centered at the Origin

\[
\left( \frac{x}{r} \right)^2 + \left( \frac{y}{r} \right)^2 = 1
\]

The unit circle is stretched \( r \) times wider and \( r \) times taller.

\[
\left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1
\]

The unit circle is stretched \( a \) times wider and \( b \) times taller.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
General Form of an Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Center at \((h, k)\)

Vertices at \((h + a, k)\), \((h - a, k)\), \((h, k + b)\), \((h, k - b)\)

Example 1

Graph $9(x - 3)^2 + 16(y + 2)^2 = 144$

$$\frac{(x - 3)^2}{16} + \frac{(y + 2)^2}{9} = 1$$

Example 2

$$\frac{(x + 2)^2}{16} + \frac{(y + 1)^2}{36} = 1$$

Recap

General Equation of an Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Center at \((h, k)\)

Vertices at \((h + a, k)\), \((h - a, k)\), \((h, k + b)\), \((h, k - b)\)