Preliminaries and Objectives

Preliminaries
- Transformation of graphs (shifting and stretching)

Objectives
- Graph a hyperbola, given the equation.
- Find the equation of a hyperbola, given the graph.

General Equation of a Hyperbola

Ellipse Centered at the Origin

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

The unit circle is stretched \( a \) times wider and \( b \) times taller.

Standard Hyperbola

\[ x^2 - y^2 = 1 \]
### Standard Hyperbola - Vertical

\[ y^2 - x^2 = 1 \]

![Diagram of a standard hyperbola with asymptotes and key points labeled: (-2, 3) and (2, 3) on the y-axis, (-2, 0) and (2, 0) on the x-axis, (-2, -3) and (2, -3) on the extended x-axis.]

### Stretched Hyperbola

The hyperbola is stretched \( a \) times wider and \( b \) times taller.

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

### Writing the equation from the graph

\[ (\_\_\_)^2 - (\_\_\_)^2 = 1 \]

![Diagram of a stretched hyperbola with asymptotes and key points labeled: (-1, 4) and (5, 4) on the y-axis, (-1, -1) and (5, -1) on the x-axis, (-1, -6) and (5, -6) on the extended x-axis.]

University of Minnesota General Equation of a Hyperbola
**Writing the equation from the graph**

$$\frac{(x - 2)^2}{9} - \frac{(y + 1)^2}{25} = 1$$

Center at $(2, -1)$

**Recap**

**General Equation of a Hyperbola - Horizontal**

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Center at $(h, k)$

Asymptotes have slope $\pm \frac{b}{a}$ and pass through the center

Vertices at $(h + a, k), (h - a, k)$

**General Equation of a Hyperbola - Vertical**

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Center at $(h, k)$

Asymptotes have slope $\pm \frac{b}{a}$ and pass through the center

Vertices at $(h, k + b), (h, k - b)$