

## Combining Trig Functions and Inverse Trig Functions - Part II



## Preliminaries and Objectives

Preliminaries:

- Trig functions
- Inverse Trig Functions
- Angle Sum Formulas

Objectives:

- Find values when trig functions and inverse trig functions are combined in angle sum formulas.

### Angle Sum Formulas - Example 1

Find

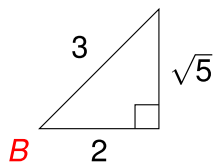
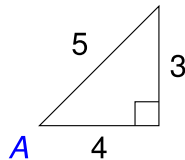
$$\sin \left( \tan^{-1} \frac{3}{4} + \cos^{-1} \frac{2}{3} \right) \Leftrightarrow \sin ( A + B )$$

$$\tan A = \frac{3}{4}; \sin A = \frac{3}{5}; \cos A = \frac{4}{5}$$

$$\cos B = \frac{2}{3}; \sin B = \frac{\sqrt{5}}{3}; \tan B = \frac{\sqrt{5}}{2}$$

$$\sin ( A + B ) = (\sin A)(\cos B) + (\cos A)(\sin B)$$

$$= \left( \frac{3}{5} \right) \left( \frac{2}{3} \right) + \left( \frac{4}{5} \right) \left( \frac{\sqrt{5}}{3} \right) = \frac{6 + 4\sqrt{5}}{15}$$

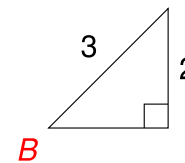
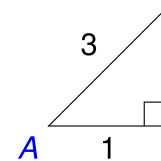


### Angle Sum Formulas - Example 2

Find

$$\cos \left( \sec^{-1} 3 + \sin^{-1} \frac{2}{3} \right) \Leftrightarrow \cos ( A + B )$$

$$\sec A = 3; \cos A = \frac{1}{3}$$



## Angle Sum Formulas - Example 3

To find

$$\csc \left( \tan^{-1} \frac{3}{4} + \cos^{-1} \frac{2}{3} \right) \Leftrightarrow \csc (A + B) \Leftrightarrow \frac{1}{\sin (A + B)}$$

first find

$$\sin \left( \tan^{-1} \frac{3}{4} + \cos^{-1} \frac{2}{3} \right) = \frac{6+4\sqrt{5}}{15}$$

then take the reciprocal to get

$$\csc \left( \tan^{-1} \frac{3}{4} + \cos^{-1} \frac{2}{3} \right) = \frac{15}{6+4\sqrt{5}} = \frac{30\sqrt{5}-45}{22}$$

## Recap

- Inverse trig functions give information about two sides of a triangle
- The third side can be found by the Pythagorean Theorem
- Values of trig functions can be found from the triangle and plugged into the appropriate formula.

This technique will also work for double and half angle formulas.

## Angle Sum Formulas - Solution to Example 2

In case you wanted the answer to the second example:

$$\cos \left( \sec^{-1} 3 + \sin^{-1} \frac{2}{3} \right) \Leftrightarrow \cos (A + B)$$

$$\sec A = 3; \cos A = \frac{1}{3}; \sin A = \frac{2\sqrt{2}}{3}$$

$$\sin B = \frac{2}{3}; \cos B = \frac{\sqrt{5}}{3}$$

$$\cos \left( \sec^{-1} 3 + \sin^{-1} \frac{2}{3} \right) = \left( \frac{1}{3} \right) \left( \frac{\sqrt{5}}{3} \right) - \left( \frac{2\sqrt{2}}{3} \right) \left( \frac{2}{3} \right) = \frac{\sqrt{5}-4\sqrt{2}}{9}$$