1. The Inverse Functions of Sine and Cosine

2. You should be familiar with values of sin and cos from the unit circle.

   In this lesson, we will define the inverse functions of sine and cosine and find some special values.

3. The sine function has as its input, an angle, and has as its output, a number between -1 and 1.

4. (a) We can create the inverse of the sine function by reversing the roles of the input and the output.

   (b) The inverse sine function is written as sine with a -1 that looks like an exponent. This is not a reciprocal, it is an inverse. y is the angle whose sine is x.

5. (a) This expression asks for the angle whose sine is $\frac{\sqrt{2}}{2}$. You may be tempted to say that 45° is the angle whose sine is $\frac{\sqrt{2}}{2}$, but that is only partly correct.

   (b) There are many angles whose sine is $\frac{\sqrt{2}}{2}$, for instance, 135°. We could go around a full circle to 405°, we could go backward to $-225°$. Which one should we pick?

6. (a) In finding an inverse to $y = x^2$, we had to make a choice as to whether we wanted the positive number or the negative number to be the square root.

   (b) We chose the positive number. It was an arbitrary choice, but we understand the square root symbol to mean the POSITIVE square root.

7. For the inverse sine, we have to choose between the right half of the circle, or the left half.

8. We will choose the right half, so that the output of the inverse sine function is always between $-90°$ and 90°.

9. Here are the common values with which you should be familiar.

10. Of course, we could also give the answers in radians, rather than degrees.

11. A similar choice is needed for the inverse cosine. For any given number between -1 and 1, there are two angles between 0° and 360° with that given number as its cosine value. One is on the top of the circle, the other is on the bottom.

12. We will choose the one on the top of the circle, so that the output of the inverse cosine function is always between 0° and 180°.

13. Here are the common values with which you should be familiar for inverse cosine.

14. And in radians.

15. To recap: The inverse of a trig function reverses the roles of the input and the output, so that the input to an inverse trig function is a number, and the output is an angle. The numbers that are input into the inverse sine and inverse cosine functions range from -1 to 1, and include the special unit circle values of $\frac{1}{2}, \frac{\sqrt{2}}{2}$ and $\frac{\sqrt{3}}{2}$. The outputs are angles. For inverse sin, the output should be on the right half of the unit circle, between $-90°$ and 90°. For inverse cosine, the output should be on the top half of the unit circle, between 0° and 180°.