1. Solving Trig Equations

2. You should be familiar with finding values and angles from the unit circle, both in degrees and radians. You should also be familiar with inverse trig functions, and with algebraic techniques to solve polynomial equations.

   In this lesson we will find all the angles that are solutions to a trigonometric equation.

3. (a) We may be asked to find angles which solve certain equations. This is related to the idea of the inverse trig functions since we will be given information about the trig values, and wish to find the angles. Here is a simple example. We can use the unit circle to find the two angles between $0°$ and $360°$ where the sine value is $-\frac{\sqrt{3}}{2}$, namely $240°$ and $300°$.

   (b) However there are more angles that are solutions. We can travel forward around the unit circle to find additional solutions.

   (c) By adding a full circle of $360°$ to our initial answers, we will get other answers. We can continue adding $360°$ as many times as we wish.

   (d) We can also subtract $360°$ to find other solutions.

4. (a) We can also do this problem in radians.

   (b) Again, we find the unit circle values of $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$, then add and subtract full circles as many times as we wish.

5. (a) We may need to use some algebra skills, like solving linear equations. $\sin\theta$ is a number, which varies depending on the input angle. Think of $\sin\theta$ as a variable like x. The equation is similar to the equation $3x - 2 = -1/2$. To solve $3x - 2 = -1/2$, we would first add 2 to both sides, then divide by 3. In solving our trig equation, we perform the same steps, first adding 2 to both sides.

   (b) Then dividing by 3.

   (c) at which point we can look up the angles from the unit circle.

   (d) Once we have found the two values from the unit circle, we can add and subtract full circles.

6. (a) This problem requires us to take a square root. $\tan\theta$ is a variable expression, so solving this equation is similar to solving $x^2 = 3$.

   (b) First we take a square root of both sides.

   (c) Then look up the unit circle solutions, and add and subtract full circles.

7. (a) This equation is similar to the equation $2x^2 + x - 1 = 0$, which we would solve by factoring.

   (b) Here we factor the polynomial

   (c) find the solutions for each factor separately

   (d) Look up the values from the unit circle

   (e) and add and subtract full circles.

8. To recap: First, think of expressions like $\sin\theta$ as variables, and use algebraic techniques to solve these equations for the trig functions. Then look up the angles from the unit circle. Finally, to get a complete set of solutions, add and subtract full circles.