1. Parametric Equations

2. You should be familiar with equations of lines, including point-slope form. You should also be familiar with the equations of circles and ellipses, and how the *sin* and *cos* functions relate to the unit circle.

   In this lesson, we will study functions and graphs where the $x-$ and $y-$coordinates are defined separately as functions of time.

3. The standard approach to graphing is the function approach. The input variable is x, and the graph moves from left to right, plotting one y-point for each x.

4. (a) Sometimes, the object we wish to graph is not a function, for example, a circle. In the function approach, we need to write two equations, one for the top half of the circle,
   (b) and one for the bottom.
   (c) .

5. (a) Another approach is to imagine the graph being drawn by a particle moving in the plane through time. We define the x and y coordinates separately as functions of time. This allows our graph to move back and forth, left and right. For the circle, we have a simple solution, beginning at the point $(1, 0)$ when time $t = 0$, we let $x = \cos t$ and $y = \sin t$.
   (b) Now as t goes from 0 to 2π,
   (c) the circle is traced in a counterclockwise direction.
   (d) In addition to the graph traced out,
   (e) in this case, a circle,
   (f) we also get information about the direction of travel,
   (g) the circle is traced
   (h) in a counterclockwise direction
   (i) at a constant speed as we move forward in time.

6. (a) Parametric equations are also convenient for ellipses. This set of parametric equations will stretch the circle 5 times wider and 3 times taller, making an ellipse. It may not be obvious at first that these equations represent an ellipse,
   (b) however, if you plug in the functions for x and y in the bottom equation, you will get a statement which is true.

7. We can also limit the time over which we trace. The first equation is the standard unit circle, starting from the positive side of the x-axis and travelling counterclockwise around the circle once.

   One technique to draw a parametric graph is to pick a few time values, plug them into the equations, and plot the points.

   In the second equation, you may think you get a different graph, but in fact we get the same graph. When $t = \pi/4$, then $2t = \pi/2$ and the x-value will be 0 and the y-value will be 1. All that has happened is that we reach the top of the circle twice as fast, The second graph is the same as the first graph, just drawn twice as fast.

The third graph is also a circle, it starts at (0,1), which is the top of the circle, and then procedes around clockwise.

The fourth graph is the standard unit circle, traced counterclockwise twice.

8. (a) Let's write parametric equations for this ellipse. We can think of this ellipse as the unit circle which we have modified by shifting and stretching. If we let $x = \cos t$ and $y = \sin t$, we will get the standard unit circle, starting at the right, and proceding counterclockwise. The center of the ellipse is at $(2, 3)$, so we can shift the x-value by 2 and the y-value by 3.
   (b) The circle of radius 1 is also stretched by a factor of 4 in the x-direction and a factor of 2 in the y-direction.
   (c) As t goes from 0 to 2π, the particle makes one trip around the ellipse counterclockwise.
9. We can also graph lines parametrically. The components are the same components in the point-slope form of a line. The coefficient on t in the x-equation is the speed in which we are moving in the x-direction and similarly for y. Divide these two numbers and we have the relative speed, which is the slope. When $t = 0$, we are at the point $(1, -2)$. We can then write the point-slope equation of the line.
10. If we restrict the time over which we trace, we get a line segment.
11. (a) To write the parametric equations of a line segment, we use the location at $t = 0$ as our starting location. x will begin at 3 and y at -2. We then move the point by determining the speed at which it moves.
    (b) In the x-direction, the point moves form 3 to -12, for a total of left 15, which happens as t changes from 0 to 5, so the x-coordinate decreases by 3 for each unit of time. Similarly, the y-coordinate goes up 2 in each unit of time.
12. (a) Given a function, there is an easy way to write the parametric equation. Simply let $x = t$, so the graph is traced from left to right at constant speed.

    Substituting x for t in the y-equation gives the equation of a standard parabola.
    (b) .
13. (a) In the other direction, given a set of parametric equations, we may wish to find the function. One technique is to solve one equation for t
    (b) then substitute that value into the other equation.
    (c) we can then simlipfy
    (d) to get the standard form of a parabola pointing to the right, with vertex at $(4, -4)$ and stretched by a factor of $3/4$.
14. To recap: Given x and y defined parametrically as functions of t, we can plot the graph by plugging in values of t to get several points, and then sketch the graph. Given y as a function of x, we can get a set of parametric equations by letting $x = t$. Given x and y as a set of parametric equations with respect to t, if we wish to express y as a function of x, one technique is to solve one equation for t and substitute into the other equation.